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A Comparative Study of Equalization Techniques for MIMO Systems

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Abstract – **Multiple-input-multiple-output (MIMO) system has become one of the major focuses in the research community of wireless communication and information theory. Due to the channel characteristics, severe inter-symbol interference (ISI) may occur to the transmitted signals. Therefore it is necessary to use an equalizer to counter the effect of the channel. The effect of fading and interference effects can be combated with equalizer. In this paper, we propose different equalization scheme for MIMO systems such as, Zero Forcing (ZF) equalization, Minimum Mean Square Error (MMSE) equalization, Zero Forcing equalization with Successive Interference Cancellation (ZF-SIC), ZF-SIC with optimal ordering , MIMO with MMSE SIC and optimal ordering and ML.**

*Keywords – **equalization techniques, MIMO system***

I. Introduction

When a signal is transmitted over a radio channel, it is subject to reflection, refraction and diffraction and also the type of modulation technique selected at transmitter. The communication environment changes quickly and thus introduce more complexity and uncertainty to channel response. OFDM is one of the best multiplexing techniques which compensate intersymbol- interference as well as co- channel- interference. In wireless Communication, scare resources and hence imposes a high cost on the high data rate transmission. Fortunately, the emergence of multiple antenna system has opened another very resourceful dimension space, for information transmission in the air.

In OFDM [1]–[2], the entire channel is divided into many narrow parallel sub-channels, thereby increasing the symbol duration and reducing or eliminating the ISI caused by the multipath. Therefore, OFDM has been used in digital audio and video broadcasting in Europe [3], and is a promising choice for future high-data-rate wireless systems. Multiple transmit and receive antennas can be used with OFDM to further improve system performance.

In recent years, multiple-input, multiple output (MIMO) wireless systems have received considerable attention due to the high data rates they provide. Orthogonal frequency division multiplexing (OFDM), a digital multi-carrier modulation technique, is well suited to be used in MIMO systems as it provides the ability to operate in frequency-selective channel environments. When OFDM is combined with the capacity increase provided by MIMO systems, the result is a very successful communication system.

Communication in wireless channels is impaired predominantly by multipath fading. Multipath is the arrival of the transmitted signal at the receiver through differing angles and/or differing time delays and/or differing frequency. MIMO offers significant increases in data throughput and link range without additional bandwidth or transmit power. It achieves this by higher spectral efficiency and link reliability and or diversity. Because of these three properties, MIMO is an important part of modern wireless communication. Due to the channel characteristics, severe inter-symbol interference (ISI) may occur to the transmitted signals. The effect of fading and interference effects can be combated with equalizer.

In this paper we have discussed different types of equalizer like ZF, MMSE, ZF-SIC, ZF-SIC and optimal ordering, MMSE-SIC and optimal ordering, ML.

We discuss here the zero-forcing (ZF) detector and the minimum-mean-square error (MMSE) detector. The ZF-detector and the MMSE detector have lower computational calculations as they require only a matrix operations to be carried out, e.g. pseudo-inverse. However, the error performance in case of both ZF and MMSE is lower as concluded from the previous papers. Our approach in this paper is to find the best equalization technique.

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II. METHODS

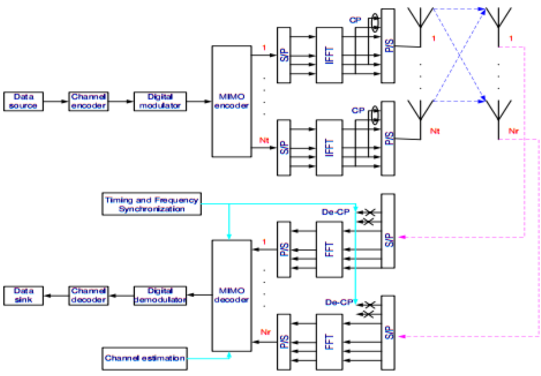

Figure 1.block diagram of MIMO-OFDM

Equation shows expression for a MIMO-OFDM system with T transmit and R receive antennas, the received signal at the k-th sub-carrier of the n-th block from the j-th receive antenna:

$$Y_i = \sum_{i=1}^{T} H_{ij}[n,k]x_i[n,k] + \omega_j[n,k]$$

for j = 1,…, R and k = 0,…, K - 1, where xi [n; k] is the symbol transmitted from the i-th transmit antenna at the k-th subcarrier of the n-th block, Hji [n; k] is the channel's frequency response at the k-th sub-carrier of the n-th block corresponding to the i-th transmit and the j-th receive antenna, and, j [n; k] is additive (complex) Gaussian noise[4] .

Zero Forcing Equalizer

Zero Forcing Equalizer is a linear equalization algorithm used in communication systems; it inverts the frequency response of the channel. The name Zero forcing corresponds to bringing down the Inter Symbol Interference (ISI) to zero in a noise free case. This will be useful when ISI is more predominant when comparing to the noise.
Consider a 2x 2 MIMO channel, the received signal on the first receive antenna is,

$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1 = [h_{11} h_{12}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

The received signal on the Second receive antenna is,

$$y_2 = h_{21}x_1 + h_{22}x_2 + n_2 = [h_{21} h_{22}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2$$

Where
y_1, y_2 : are the received symbols on the first and second antenna respectively, h_{11} is the channel from 1-st transmit antenna to 1-streceive antenna, h_{12} is channel from 2-nd transmit antenna to 1-st receive antenna, h_{21} is the channel from 1-st transmit antenna to 2-nd receive antenna, h_{22} is the channel from 2-nd transmit antenna to 2-nd receive antenna, x_1, x_2 are the transmitted symbols and n_1, n_1 are the noise on 1-st and 2-nd receive antennas.
The equation can be represented in matrix notation as follows:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

Equivalently, Y = HX + N 1.4

Where, Y = Received Symbol Matrix. H = Channel matrix. X = Transmitted symbol Matrix. N = Noise Matrix.

To solve for x, we need to find a matrix W which satisfies WH = I. The Zero Forcing (ZF) detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H$$

Where
W - Equalization Matrix and H - Channel Matrix
This matrix is known as the Pseudo inverse for a general m x n matrix where

$$H^H H = \begin{pmatrix} h_{11}^* & h_{21}^* \\ h_{12}^* & h_{22}^* \end{pmatrix} \begin{pmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{pmatrix}$$
$$= \begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 & h_{11}^* h_{12} + h_{21}^* h_{22} \\ h_{12}^* h_{11} + h_{22}^* h_{21} & |h_{12}|^2 + |h_{22}|^2 \end{bmatrix}$$

Zero forcing equalizer tries to null out the interfering terms when performing the equalization while doing so, there can be amplification of noise. Hence Zero forcing equalizer is not the best possible equalizer.

MMSE Equalizer

The MMSE approach tries to find a coefficient W which minimize the criterion,

$$E\left\{[W_{y-x}][W_{y-x}]^H\right\}$$

Solving

$$W = (H^H H + N_0 I)^{-1} H^H$$

When comparing to the equalization, apart from the $N_0 I$ team both the equations are comparable. When the noise term is zero, the MMSE equalization reduced to ZF equalizer [5].

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Zero Forcing with Successive Interference Cancellation (ZF-SIC)

Using the Zero Forcing (ZF) equalization approach described above, the receiver can obtain an estimate of the two transmitted symbols x_1, x_2 i.e.

$$\begin{bmatrix}\widehat{x_1}\\ \widehat{x_2}\end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix}y_1\\ y_2\end{bmatrix}$$

Take one of the estimated symbols (for example $\widehat{x_1}$) and subtract its effect from the received vector y_1 and y_2 , i.e.

$$\begin{bmatrix}r_1\\ r_2\end{bmatrix} = \begin{bmatrix}y_1 - h_{12}\widehat{x_1}\\ y_2 - h_{22}\widehat{x_2}\end{bmatrix} = \begin{bmatrix}h_{11}x_1 + n_1\\ h_{21}x_2 + n_2\end{bmatrix}$$

Expressing in matrix notation,

$$\begin{bmatrix}r_1\\ r_2\end{bmatrix} = \begin{bmatrix}h_{11}\\ h_{21}\end{bmatrix} x_1 + \begin{bmatrix}n_1\\ n_2\end{bmatrix}$$

$$r = hx_1 + n$$

The above equation is same as equation obtained for receive diversity case. Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply Maximal Ratio Combining (MRC).

The equalized symbol is,

$$\widehat{x_1} = \frac{h^H r}{h^H h}$$

This forms the simple explanation for Zero Forcing Equalizer with Successive Interference Cancellation (ZF-SIC) approach.

ZF-SIC with optimal ordering

In classical Successive Interference Cancellation, the receiver arbitrarily takes one of the estimated symbols, and subtract its effect from the received symbol y_1 and y_2. However, we can have more intelligence in choosing whether we should subtract the effect of $\widehat{x_1}$ first or $\widehat{x_2}$ first. To make that decision, let us find out the transmit symbol (after multiplication with the channel) which came at higher power at the receiver. The received power at the both the antennas corresponding to the transmitted symbol x_1 is,

$$Px_1 = |h_{11}|^2 + |h_{21}|^2$$

The received power at the both the antennas corresponding to the transmitted symbol x_2 is,

$$Px_2 = |h_{12}|^2 + |h_{22}|^2$$

If $Px_1 > Px_2$ then the receiver decides to remove the effect of $\widehat{x_1}$ from the received vector y_1 and y_2 and then re-estimate $\widehat{x_2}$.

$$\begin{bmatrix}r_1\\ r_2\end{bmatrix} = \begin{bmatrix}y_1 - h_{11}\widehat{x_1}\\ y_2 - h_{12}\widehat{x_1}\end{bmatrix} = \begin{bmatrix}h_{12}x_2 + n_1\\ h_{22}x_2 + n_2\end{bmatrix}$$

Expressing in matrix notation,

$$\begin{bmatrix}r_1\\ r_2\end{bmatrix} = \begin{bmatrix}h_{12}\\ h_{22}\end{bmatrix} x_2 + \begin{bmatrix}n_1\\ n_2\end{bmatrix}$$

$$r = hx_2 + n$$

Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply Maximal Ratio Combining (MRC). The equalized symbol is,

$$\widehat{\widehat{x_2}} = \frac{h^H r}{h^H h}$$

If $Px_1 \le Px_2$ then the receiver decides to remove the effect of $\widehat{x_2}$ from the received vector y_1 and y_2 and then re-estimate $\widehat{x_1}$.

$$\begin{bmatrix}r_1\\ r_2\end{bmatrix} = \begin{bmatrix}y_1 - h_{12}\widehat{x_2}\\ y_2 - h_{22}\widehat{x_2}\end{bmatrix} = \begin{bmatrix}h_{11}x_1 + n_1\\ h_{21}x_1 + n_2\end{bmatrix}$$

Expressing in matrix notation,

$$r = hx_1 + n$$

Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply Maximal Ratio Combining (MRC). The equalized symbol is,

$$\widehat{\widehat{x_1}} = \frac{h^H r}{h^H h}$$

Doing successive interference cancellation with optimal ordering ensures that the reliability of the symbol which is decoded first is guaranteed to have a lower error probability than the other symbol. This results in lowering the chances of incorrect decisions resulting in erroneous interference cancellation. Hence gives lower error rate than simple successive interference cancellation.

MMSE SIC and optimal ordering

It is known that SIC with optimal ordering improve the performance with ZF equalization. The concept of SIC is extended to the MMSE equalization. The MMSE approach tries to find a coefficient W which minimizes the criterion,

$$E\left\{[W_{y-x}][W_{y-x}]^H\right\}$$

$$W = (H^H H + N_0 I)^{-1} H^H$$

Solving,

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Using MMSE equalization, the receiver can obtain an estimate of the two transmitted symbol x_1, x_2 i.e.

$$\begin{bmatrix}\widehat{x_1}\\ \widehat{x_2}\end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix}y_1\\ y_2\end{bmatrix}$$

SIC with optimal ordering is however can have more intelligence in choosing whether we should subtract the effect of $\widehat{x_1}$ first or $\widehat{x_2}$ first. To make the decision, find out the transmit symbol which came at higher power at the receiver. The received power at the both the antenna corresponding to the transmitted symbol x_1 is $Px_1 = |h_{11}|^2 + |h_{21}|^2$. The received power at the both the antenna corresponding to the transmitted symbol x_2 is $Px_2 = |h_{12}|^2 + |h_{22}|^2$. If $Px_1 > Px_2$ then the receiver decides to remove the effect of $\widehat{x_1}$ from the received vector y_1 and $y_2 x_2$. Else if $Px_1 \le Px_2$ then the receiver decides to subtract the effect of x_2 from the received vector y_1 and y_2 and then re-estimate $\widehat{x_1}$ .once the effect of either $\widehat{x_1}$ or $\widehat{x_2}$ is removed, the new channel becomes a 1×2 receive antenna case and the symbol on other spatial dimension can be optimally equalized by MRC.

MIMO with ML Equalization

ML provides the better performance then MMSE equalization with optimally ordered SIC. We assume the ML for flat fading Rayleigh multipath channel and modulation in BPSK.

The ML Receiver tries to find x which minimizes, $j = |y - Hx|^2$

$$\left(\begin{pmatrix}y_1\\ y_1\end{pmatrix} - \begin{pmatrix}h_{11} & h_{12}\\ h_{21} & h_{22}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}\right)^2$$

Since the modulation is BPSK the possible values of x_1 is +1 or -1. Similarly x_2 also take values +1 or -1. So, to find the maximum likelihood solution, we need to find the minimum from the all four combination of x_1 and x_2 .The estimate of the transmit symbol is chosen based on minimum value from the above four values i.e. if the minimum is $J_{+1,+1} = [11]$ if the minimum is $J_{-1,+1} = [01]$, if the minimum is $J_{+1,-1} = [10]$, $J_{-1,-1} = [00]$[5].

III. Simulation and Results

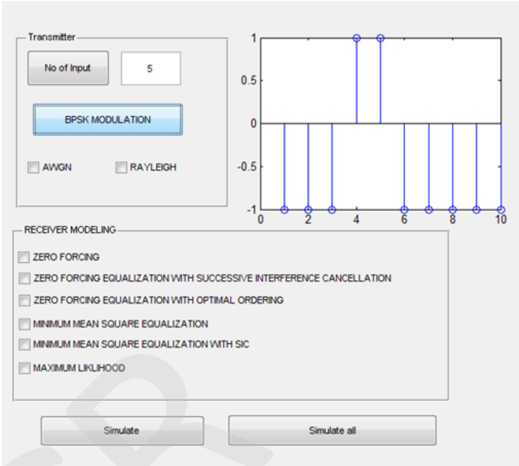

Figure 2.GUI of system

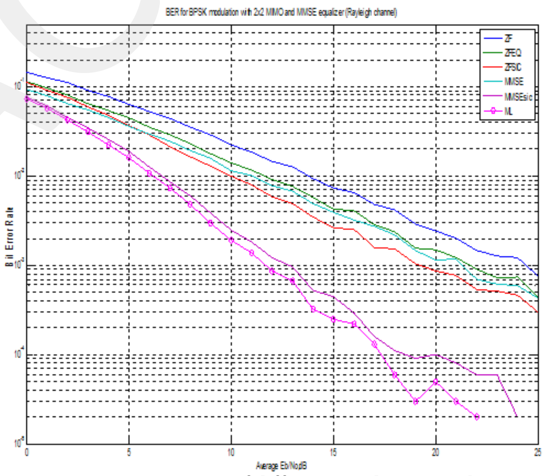

Figure 3.comparision of different equalization tech.

We find that ML is best equalization technique since it has minimum BER for given channel parameter.

IV. Conclusion

We proposed the MIMO system for the detection of the symbol with minimum BER. Assumed that the channel is a flat fading Rayleigh multipath channel and the modulation is BPSK. We discussed the 2x2 MIMO system for different equalization techniques. We studied that the MIMO receiver with ML (maximum likelihood) detection has the least BER for a given SNR.

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