

# Transmit Diversity Tradeoff between Spectral Efficiency in Spatial Modulation-STBC with ZF, MMSE, and Sphere Decoder

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*Abstract* - Transmit diversity is easier to achieve by STBC but at the same time it is not feasible to define spectral efficiency, so achieve the tradeoff between that spatial modulation with STBC has been proposed in this paper. In this paper a novel detection algorithm for spatial modulation (SM) based on different receiving algorithm; ZF (Zero Forcing), MMSE (minimum mean-square error), and Sphere Decoder is proposed. The aim is to reduce the receiver complexity of the existing optimal decoder while maintaining an optimum performance.

*Keywords* - Spatial Modulation (SM), Zero Forcing (ZF), MMSE, Sphere decoder.

## I. INTRODUCTION

MIMO Spatial Multiplexing (SM) seems to be the greatest remedy to improve the program potential without demanding extra spectral sources. Within SM, a multiple signals are usually transmitted instantly by means of enough spaced- antennas. At the receiver part, the leading problem is located inside the detection techniques, capable of separating those transmitted signals with acceptable complexity and achieved performance.

Higher data rate and better spectral efficiency are of paramount importance in next generation cellular communication. MIMO (multiple input multiple output) technology is approach to attain high spectral efficiency by transmitting multiple data streams from multiple antennas. MIMO technology is an effective means to solve the conflict of future high speed wireless communications and the limited spectrum resources can be used [1].

A multiple-input multiple-output (MIMO) system that exploits this potential is the V-BLAST (Vertical Bell Labs Layered Space-Time) architecture proposed in [3]. It uses a vertically layered coding structure, where independent code blocks (called layers) are associated with a particular transmit antenna. At the receiver, these layers are detected by a successive interference cancellation technique which nulls the interferers by linearly weighting the received signal vector with a zero-forcing nulling vector (ZF-BLAST). There are two main problems for VBLAST system; one is high inter-channel interference (ICI) at the receiver due to simultaneous transmissions on the same frequency from multiple antennas, which requires a complex receive algorithm. The other is the number of receive antennas must be greater or equal to the number of the transmit antennas. Spatial modulation (SM) technique proposed by Mesleh [4] can overcome the above problems. SM is an extension of conventional modulation, and the constellation with 3 dimensions is constructed. A very efficient detection algorithm utilizing the QR decomposition of the channel matrix was proposed by the authors in [5], [6]. It jointly calculates an optimized detection order and the QR decomposition of the channel matrix and is called ZF-SQRD (ZF Sorted QR Decomposition). An adaption of the original ZF-BLAST to the MMSE criterion was presented in [7] and a version with lower complexity was introduced in [8]. Furthermore, among the possible design criteria, we retain the joint transmit and receive minimum mean squared error (joint Tx/Rx MMSE) criterion, initially proposed in [9] and further discussed in

[10] and [11], for it is the optimal linear solution for fixed coding and modulation across the spatial streams.

## II. SYSTEM MODEL AND ALGORITHM

### Linear Detection Techniques

The idea behind linear detection techniques is to linearly filter received signals using filter matrices, as depicted in Figure1. This category includes Zero-Forcing (ZF), Minimum Mean Square Error (MMSE) and Sphere Decoder techniques. Although linear detection schemes are easy to implement, they lead to high degradation in the achieved diversity order and error performance due to the linear filtering.

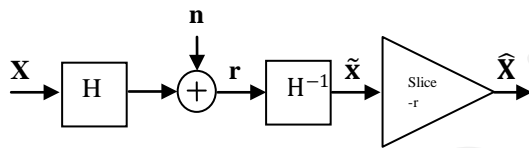


Figure1: MIMO SM with linear receiver

### Zero-Forcing

Zero-Forcing (ZF) technique is the simplest MIMO detection technique, which was proposed in [3]. Where filtering matrix is constructed using the ZF performance based criterion. The drawback of ZF scheme is the susceptible noise enhancement and loss of diversity order due to linear filtering [12], [13]. ZF can be implemented by using the inverse of the channel matrix  $H$  to produce the estimate of transmitted vector  $\tilde{x}$

$$\begin{aligned}\tilde{x} &= H^\dagger r \\ &= H^\dagger (Hx) \\ &= x\end{aligned}$$

Where  $(.)^\dagger$  denotes the pseudo-inverse. But when the noise term is considered, the post-processing signal is given by:

$$\begin{aligned}\tilde{x} &= H^\dagger R \\ &= H^\dagger (Hx+n) \\ &= x + H^\dagger n\end{aligned}$$

With the addition of the noise vector, ZF estimate, i.e.  $\tilde{x}$  consists of the decoded vector  $x$  plus a combination of the inverted channel matrix and the

unknown noise vector. Because the pseudo-inverse of the channel matrix may have high power when the channel matrix is ill-conditioned, the noise variance is consequently increased and the performance is degraded. To alleviate for the noise enhancement introduced by the ZF detector, the MMSE detector was proposed, where the noise variance is considered in the construction of the filtering matrix  $G$ .

### Minimum Mean Square Error

Minimum Mean Square Error (MMSE) approach alleviates the noise enhancement problem by taking into consideration the noise power when constructing the filtering matrix using the MMSE performance-base criterion. The vector estimates produced by an MMSE filtering matrix becomes

$$\tilde{x} = [(H^H H + (\sigma^2 I))^{-1} H^H] r \quad (1)$$

Where  $\sigma^2$  is the noise variance. The added term ( $1/SNR = \sigma^2$ , in case of unit transmit power) offers a trade-off between the residual interference and the noise enhancement. Namely, as the SNR grows large, the MMSE detector converges to the ZF detector, but at low SNR it prevents the worst Eigen values from being inverted. At low SNR, MMSE becomes Matched Filter

$$[(H^H H + (\sigma^2 I))^{-1} H^H] \approx \sigma^2 H^H \quad (2)$$

At high SNR, MMSE becomes ZF:

$$[(H^H H + (\sigma^2 I))^{-1} H^H] \approx (H^H H)^{-1} H^H \quad (3)$$

### Sphere decoder (SD)

Techniques like Maximum likelihood decoding require an exhaustive search over all possible code words used. The computational complexity of these techniques is exponential in the length of the codeword. A promising approach called the sphere decoding algorithm was proposed to lower the computational complexity. The principle of the sphere decoding algorithm is to search the closest lattice point to the received signal within a sphere radius, where each codeword is represented by a lattice point in a lattice field. In 2-D the search can be represented by drawing a circle around the received signal just small enough to enclose one lattice point and eliminate the search of other points outside the circle as shown in Figure2 below:

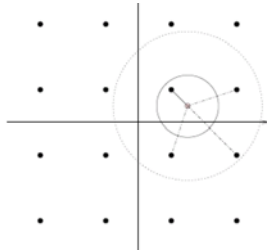


Figure2: Geometrical representation of the sphere decoding algorithm

As stated in the introduction, the exhaustive search through the whole lattice  $\{Hs\}$  has an exponential computational complexity. This complexity is unrealizable and thus defines a bottleneck in the practical implementation of the MIMO SM systems. The SD algorithms can solve the ML (Maximum Likelihood) detection problem in (4) by searching over a restricted subset  $\Omega$  that at least contain the ML solution.

$$\tilde{\mathbf{x}} = \arg \min_{\mathbf{x}_k \in \Omega} \|\mathbf{r} - \mathbf{H}\mathbf{x}_k\|^2 \quad (4)$$

$$\mathbf{x}_k \in \Omega$$

To describe the conventional sphere decoding algorithm, consider the QR-decomposition of the channel matrix, i.e.  $\mathbf{H} = \mathbf{Q}\mathbf{R}$  where  $\mathbf{R}$  is upper triangular matrix and  $\mathbf{Q}$  has orthogonal columns of unit norm. In the basis given by the columns of  $\mathbf{Q}$  the system model can equivalently be written as

$$\mathbf{y} = \mathbf{R}\mathbf{x} + \mathbf{v} \quad (5)$$

Where  $\mathbf{y} = \mathbf{Q}^H \mathbf{x}$  and  $\mathbf{v} = \mathbf{Q}^H \mathbf{n}$ . Further, the ML detection problem can equivalently be written as

$$\tilde{\mathbf{x}} = \arg \min_{\mathbf{x}_k \in \Omega} \|\mathbf{y} - \mathbf{R}\mathbf{x}_k\|^2 \quad (6)$$

$$\mathbf{x}_k \in \Omega$$

The main difference between (6) and (4) is that  $\mathbf{R}$ , by construction, is upper triangular. The sphere decoder solves (6) by searching over all vectors,  $\mathbf{x}_k \in \Omega$ , satisfying a spherical constraint on the form

$$\|\mathbf{y} - \mathbf{R}\mathbf{x}_k\|^2 \leq d^2 \quad (7)$$

In what follows,  $d$  will be referred to as the search radius for the obvious reason. It is straightforward to see that if  $d$  is sufficiently large, at least one vector,  $\mathbf{x}_k \in \Omega$  satisfies (7) and the SD algorithm will obtain the ML estimate. Naturally, it

is not practically feasible to verify (7) for every  $\mathbf{x}_k \in \Omega$  as this would require an exhaustive computational complexity equal to the original brute-force ML approach. Instead, SD algorithm finds all  $\mathbf{x}_k \in \Omega$  satisfying (7) through a constrained tree search.

So far, note that

$$\|\mathbf{y} - \mathbf{R}\mathbf{x}\|^2 = \sum_{i=1}^m |x_i - \sum_{j=1}^m |r_{ij}x_j|^2 \leq d^2 \quad (8)$$

Where  $r_{ij}$ ,  $y_i$  and  $x_i$  are the  $(i, j)^{\text{th}}$  entry of  $\mathbf{R}$ ,  $i^{\text{th}}$  entry of  $\mathbf{y}$  and the  $j^{\text{th}}$  entry of  $\mathbf{x}$  respectively. Thus, a sufficient condition for (7) to be satisfied is given by (9),

$$\sum_{i=m-k+1}^m |x_i - \sum_{j=1}^m |r_{ij}x_j|^2 \leq d^2 \quad (9)$$

### III. SIMULATION RESULTS

Our design has shown that by varying transmit antenna then BER performance is better in figure 4. In Figure 5 we changed the antenna combination and found that  $4 \times 4$  has given better BER then other one. Figure 3 shows the BER performance for different receiving algorithm, ZF, MMSE, SD. We have compared SD with ML and found that SD performance is near to ML which is best amongst, ZF, MMSE.

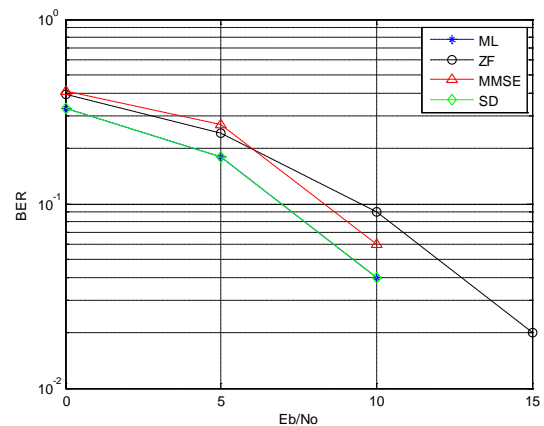


Figure3: Different receiving with ZF, MMSE, and SD

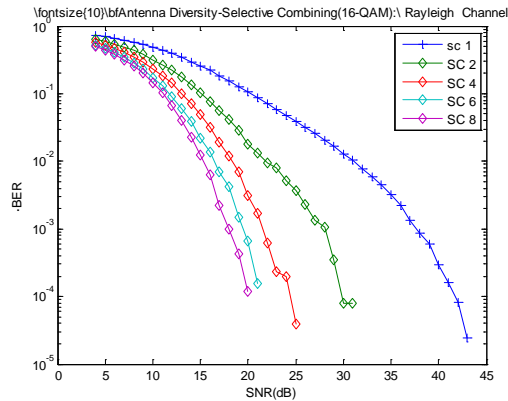


Figure4: BER under different Transmit Antenna

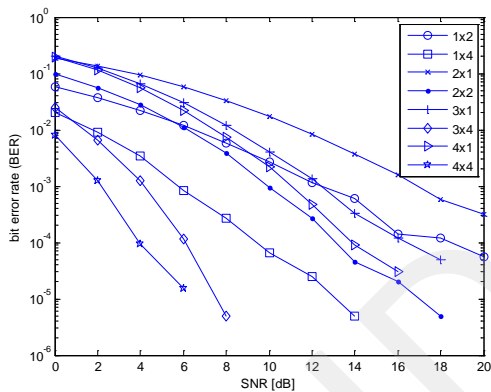


Figure5: BER under different transmit and receive combinations

#### IV. CONCLUSION

We have implemented STBC-SM for different transmit antenna configuration to achieve the transmit diversity. At the other hand by using spatial modulation we have achieved spectral efficiency, hence it is a trade of transmit diversity and spectral efficiency both. We have implemented ZF, MMSE, and SD equalizer in STBC and compare the BER performance then found that Sphere Decoder is better than MMSE, ZF. For different transmit antenna we found that as we increase the number of antenna it give better BER performance.

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