

Transmit Diversity using Various STBC Coding for MIMO Systems

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Abstract— This paper shows the performance analysis of Bit Error Rate (BER) in MIMO system using STBC codes. We compare Toeplitz OSTBC scheme with different coding schemes viz; ABBA, Jafarkhani, and Tarokh. Zero Forcing (Z-F), Minimum Mean Square Error (MMSE) equalization techniques are used for achieve full diversity.

Keywords- BER, MIMO, ABBA, ZF, MMSE.

I. INTRODUCTION

An improving demand for services for the wireless systems has turned wide variety into a valuable resource. Multiple-Input multiple-output (MIMO) wireless channels have considerably higher capacities than traditional channels. Fading makes it extremely difficult for the receiver to recover the transmitted signal unless the receiver is provided with some form of diversity, i.e. replicas of the same transmitted signal with uncorrelated attenuation. In fact, diversity combining technology has been one of the most important contributors to reliable wireless communications. Consider transmit diversity by deploying multiple antennas at the base station. Moreover, in economic terms, the cost of multiple transmit antennas at the base station can be amortized over numerous mobile users. Hence transmit diversity has been identified as one of the key contributing technologies to the downlinks of 3G wireless systems such as W-CDMA and CDMA2000. There are generally three categories of transmit diversity:

Feedback Scheme:

This involves the feedback of channel state information (CSI, typically including channel gain and phase information) from the receiver to the transmitter in order to adapt the transmitter to the channel during the next transmission epochs. It is also commonly known as the “closed-loop” system.

Feed-forward Scheme:

This involves the receiver making use of feed-forward information sent by the transmitter, such as

pilot symbols, to estimate the channel, but no channel feedback information is sent back to the transmitter. It is also commonly known as the “open-loop” or “coherent” system.

Blind Scheme:

This requires no feedback of CSI or feed-forward of pilots, and the receiver simply makes use of the received signal to attempt data recovery without the knowledge of CSI. It is also commonly known as the “non-coherent” system.

Space-Time Coding (STC) is a technique that combines coding, modulation and signal processing to achieve transmit diversity. The first STC proposed in the literature is Space-Time Trellis Code (STTC), which has a good decoding performance but decoding complexity that increases exponentially with the transmission rate. In addressing the issue of decoding complexity of STTC, Space-Time Block Code (STBC) was subsequently proposed. Alamouti [1] discovered a remarkable STBC scheme for two transmit antennas. This scheme supports linear decoding complexity for maximum-likelihood (ML) decoding, which is much simpler than the decoding of STTC. Alamouti’s scheme is very appealing in terms of implementation simplicity. Hence it motivates a search for similar schemes for more than two transmit antennas, to achieve diversity level higher than two. As a result, Orthogonal Space-Time Block Code (O-STBC) was introduced by Tarokh [2]. O-STBC is generalizations of the Alamouti’s scheme to arbitrary number of transmit antennas. It retains the property of having linear maximum-likelihood decoding with full transmit diversity.

In this paper we analyze the different space time coding style for transmit diversity of MIMO system and related receiving algorithms (Z-F and MMSE). The paper is organized as follows: Section 2 describes the Alamouti STBC scheme, ABBA, Jafarkhani, Tarokh, Section 3 represents the equalization techniques- Z-F, and MMSE. Section

4 presents simulation results. The conclusions are offered in Section 5.

II. SPACE TIME BLOCK CODES

Alamouti STBC

It is simple method for achieving spatial diversity with two transmit antennas. The scheme is as follows:

Consider that we have a transmission sequence, for example

$$\{x_1, x_2, x_3, \dots, x_n\} \quad (1)$$

In normal transmission, we will be sending x_1 in the first time slot, x_2 in the second time slot, x_3 and so on.

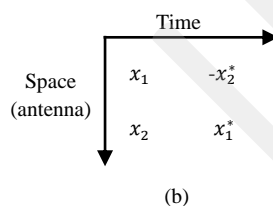
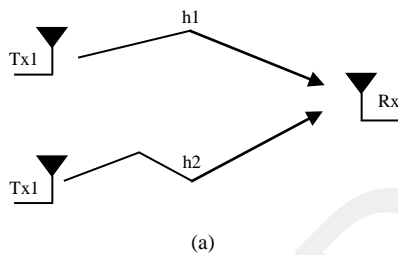


Figure1: (a) Transmit, (b) Receive Alamouti STBC coding

However, Alamouti suggested that we group the symbols into groups of two. In the first time slot, send x_1 and x_2 from the first and second antenna. In second time slot send $-x_2^*$ and x_1^* from the first and second antenna. In third time slot send x_3 and x_4 from the first and second antenna. In fourth time slot, send $-x_4^*$ and x_3^* from the first and second antenna and so on.

Notice that though we are grouping two symbols, we still need two time slots to send two symbols. Hence, there is no change in the data rate. This forms the simple explanation of the transmission scheme with Alamouti Space Time Block coding.

Alamouti scheme is an example of a full-rate full-diversity complex space-time block code.

ABBA Code

Two Alamouti codes for two transmit antennas are used as building blocks of the ABBA code for 4 transmit antennas:

$$S_{ABBA} = \begin{bmatrix} S_1 & S_2 \\ S_2 & S_1 \end{bmatrix} \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ -s_4^* & s_3^* & -s_2^* & s_1 \end{bmatrix}$$

The equivalent virtual channel matrix H_{ABBA} results in:

$$H_{ABBA} = \begin{bmatrix} H_{V1} & H_{V2} \\ H_{V2} & H_{V1} \end{bmatrix} \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ h_4^* & -h_3^* & h_2^* & -h_1 \end{bmatrix}$$

Applying a matched filter H^H at the receiver, the non-orthogonality of the ABBA code shows up in the Gramian matrix G_{ABBA} :

$$G_{ABBA} = H_{ABBA}^H H_{ABBA} = h^2 \begin{bmatrix} 1 & 0 & X & 0 \\ 0 & 1 & 0 & X \\ X & 0 & 1 & 0 \\ 0 & X & 0 & 1 \end{bmatrix}$$

Where $h^2 = h_1^2 + h_2^2 + h_3^2 + h_4^2$ is the channel is gain and X is a channel dependent interference parameter:

$$X_{ABBA} = \frac{2Re(h_1 h_3^* + h_2 h_4^*)}{h^2}$$

One major limitation of the ABBA code is its loss of robustness in highly correlated channels, especially in the case when $h_1 = h_2 = h_3 = h_4$ which leads to the collapse of all detection / decoding algorithms.

Jafarkhani Scheme

A lot of effort had been made to find orthogonal designs with highest rates in a systematic way. Therefore to reach a higher rate, one should change the structure of the orthogonal code, e.g. relaxing one of the properties of an orthogonal design. For example we can think of designing a full-diversity rate one code that does not have the property of single maximum likelihood decoding. Increasing the order of complexity of decoding by one from

separate decoding we get to pair-wise decoding; meaning that each two symbols should be detected independent of other pairs. The first such design was offered by Jafarkhani in [3] as the following matrix.

$$C = \begin{bmatrix} A & B \\ -\bar{B} & \bar{A} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix}$$

Where

$$A = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, B = \begin{bmatrix} x_3 & x_4 \\ -x_4^* & x_3^* \end{bmatrix}$$

The minimum rank of the A(C, E) is two for $C \neq E$ matrices. The design is called a Quasi Orthogonal Space-Time Block Code. The reason is that in the new design each column of the generator matrix is orthogonal to all the others except one, and so the name chosen Orthogonal.

Tarokh Scheme

A space-time block code is defined by a $m \times n$ transmission matrix G_T . The entries of the matrix G_T are linear combinations of the variables x_1, x_1, \dots, x_k and their conjugates. The number of transmission antennas is n and we usually use it to separate different codes from each other. For example, G_{T_2} represents a code which utilizes two transmit antennas and is defined by:

$$G_{T_2} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$$

We assume that transmission at the baseband employs a signal constellation A with 2^b elements. At time slot 1, kb bits arrive at the encoder and select constellation signals s_1, s_2, \dots, s_k . Setting $x_i = s_i$ for $i = 1, 2, \dots, k$ in G_T we arrive at a matrix C with entries linear combinations of s_1, s_2, \dots, s_k and their conjugates. So, while G_T contains in-determinates x_1, x_1, \dots, x_k C contains specific constellation symbols (or their linear combinations) which are transmitted from n antennas for each kb bits as follows. If C_t^i represents the element in the t^{th} row and the i^{th} column of C , the entries $C_t^i, i = 1, 2, \dots, n$ are transmitted simultaneously from transmit antennas $1, 2, \dots, n$ at each time slot $1, 2, \dots, m$. So, the i^{th} column of C represents the transmitted symbols

from the i^{th} antenna and the t^{th} row of C represents the transmitted symbols at time slot t . Note that C is basically defined using G_T , and the orthogonality of G_T 's columns allows a simple decoding scheme which will be explained in the sequel. Since m time slots are used to transmit k symbols, we define the rate R of the code to be $R = k/p$. For example, the rate of G_{T_2} is one.

In this work, we consider the performance of the following rate half space-time block codes:

$$G_{T_3} = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \\ x_1^* & x_2^* & x_3^* \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & x_4^* & x_1^* \\ -x_4^* & -x_3^* & x_2^* \end{bmatrix}$$

And

$$G_{T_4} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix}$$

Proposed Scheme

Our proposed space-time block code is defined by a $m \times n$ Toeplitz OSTBC matrix T . The entries of the matrix T are linear combinations of the variables x_1, x_1, \dots, x_k and their conjugates. T Represents a code which utilizes two transmit antennas and is defined by:

$$T = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_2^* & -x_1^* & x_4^* & -x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_2^* & -x_1^* \end{bmatrix}$$

III. RECEIVING ALGORITHM

Zero-Forcing

Zero-Forcing (ZF) technique is the simplest MIMO detection technique, Where filtering matrix is constructed using the performance based criterion. The drawback of ZF scheme is the susceptible noise enhancement and loss of diversity order due to linear filtering [4],

[5]. ZF can be implemented by using the inverse of the channel matrix H to produce the estimate of transmitted vector \tilde{x}

$$\begin{aligned}\tilde{x} &= H^\dagger r \\ &= H^\dagger(Hx) \\ &= x\end{aligned}$$

Where $(.)^\dagger$ denotes the pseudo-inverse. But when the noise term is considered, the post-processing signal is given by:

$$\begin{aligned}\tilde{x} &= H^\dagger R \\ &= H^\dagger(Hx+n) \\ &= x + H^\dagger n\end{aligned}$$

With the addition of the noise vector, ZF estimate, i.e. \tilde{x} consists of the decoded vector x plus a combination of the inverted channel matrix and the unknown noise vector. Because the pseudo-inverse of the channel matrix may have high power when the channel matrix is ill-conditioned, the noise variance is consequently increased and the performance is degraded. To alleviate for the noise enhancement introduced by the ZF detector, the MMSE detector was proposed, where the noise variance is considered in the construction of the filtering matrix G .

Minimum Mean Square Error

Minimum Mean Square Error (MMSE) approach alleviates the noise enhancement problem by taking into consideration the noise power when constructing the filtering matrix using the MMSE performance-base criterion. The vector estimates produced by an MMSE filtering matrix becomes

$$\tilde{x} = [(H^H H + (\sigma^2 I))^{-1} H^H] r \quad (5)$$

Where σ^2 is the noise variance. The added term $(1/SNR = \sigma^2, \text{ in case of unit transmit power})$ offers a trade-off between the residual interference and the noise enhancement. Namely, as the SNR grows large, the MMSE detector converges to the ZF detector, but at low SNR it prevents the worst Eigen values from being inverted. At low SNR, MMSE becomes Matched Filter

$$[(H^H H + (\sigma^2 I))^{-1} H^H] \approx \sigma^2 H^H \quad (6)$$

At high SNR, MMSE becomes ZF:

$$(H^H H + (\sigma^2 I))^{-1} H^H \approx (H^H H)^{-1} H^H \quad (7)$$

IV. SIMULATION RESULTS

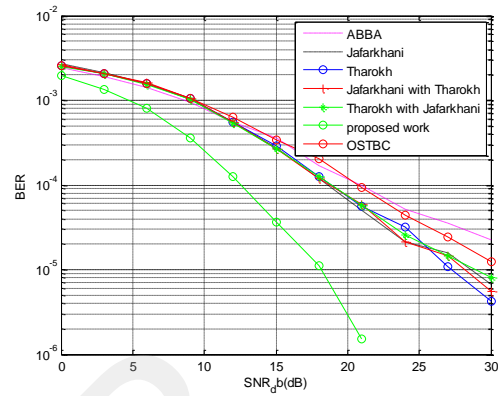


Figure 2: Variation in BER for different coding scheme

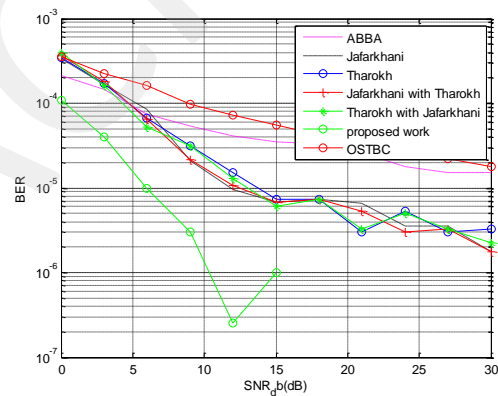


Figure 3: BER analysis for different coding scheme

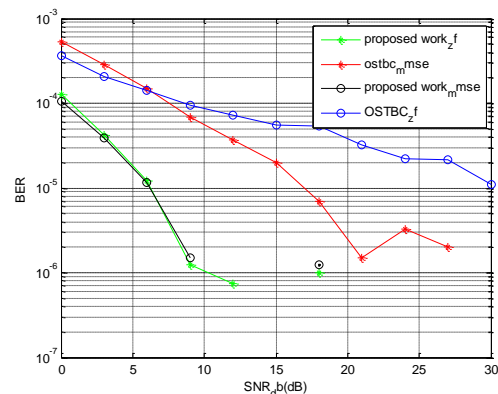


Figure 4: BER analysis for equalization techniques

V. CONCLUSION

To achieve transmit diversity, various STBC codes has been implemented. Our proposed code achieves full diversity under the equalization techniques: Zero Forcing (Z-F) and Minimum Mean Square Error (MMSE). BER measurement shows the performance analysis of MIMO system is better in MMSE equalization.

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