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A Review on Higher Order Statistics on Different Channels in Communication System

Doyal Rawal
M. Tech. Scholar, Elect. & Comm. Dept., IES, IPS Academy, Indore (India)

Mrs. Smita Patil
Asst. Professor, Elect. & Comm. Dept., IES, IPS Academy, Indore (India)

Rupesh Dubey
Associate Prof. & HOD
Elect. & Comm. Dept., IES, IPS Academy, Indore (India)

Abstract – **This paper shows a review on higher order statistics on different channels in communication system. We derive closed-form expressions of the maximal spectral efficiency over Rayleigh, Rician, and Nakagami-m multipath fading channel under flat fading conditions.**

Keywords – **Rayleigh, Rician, and Nakagami-m.**

I. Introduction

The wireless radio channel poses a severe challenge as a medium for reliable high-speed communication. It is not only susceptible to noise, interference, and other channel impediments, but these impediments change over time in unpredictable ways due to user movement. We will characterize in brief the variation in received signal power over distance due to path loss and shadowing. Path loss is caused by dissipation of the power radiated by the transmitter as well as effects of the propagation channel. Path loss models generally assume that path loss is the same at a given transmit receive distance. Shadowing is caused by obstacles between the transmitter and receiver that attenuate signal power through absorption, reflection, scattering, and diffraction. When the attenuation is very strong, the signal is blocked. Variation due to path loss occurs over very large distances (100-1000 meters), whereas variation due to shadowing occurs over distances proportional to the length of the obstructing object (10-100 meters in outdoor environments and less in indoor environments). Since variations due to path loss and shadowing occur over relatively large distances, this variation is sometimes referred to as large-scale propagation effects. In this paper we will deal with variation due to the constructive and destructive addition of multipath signal components. Variation due to multipath occurs over very short distances, on the order of the signal wavelength, so these variations are sometimes referred to as small-scale propagation effects.

II. Rayleigh Multipath Fading Model

Let the transmit band-pass signal be,

$$x(t) = \Re\{x_b(t)e^{j2\pi f_c t}\} \qquad (1)$$

Where, $x_b(t)$ is the baseband signal, f_c is the carrier frequency and t is the time.
As shown above, the transmit signal reaches the receiver through multiple paths where the n^{th} path has an attenuation $\propto_n(t)$ and delay $\tau_n(t)$. The received signal is,

$$r(t) = \sum_n \propto_n(t)x[t-\tau_n(t)] \qquad (2)$$

Plugging in the equation for transmit baseband signal from the above equation,

$$r(t) = \Re\left\{\sum_n \propto_n(t)x_b[t-\tau_n(t)]e^{j2\pi f_c[t-\tau_n(t)]}\right\} \qquad (3)$$

The baseband equivalent of the received signal is,

$$r_b(t) = \sum_n \propto_n(t)e^{-j2\pi f_c\tau_n(t)}x_b[t-\tau_n(t)] = \sum_n \propto_n(t)e^{-j\theta_n(t)}x_b[t-\tau_n(t)] \qquad (4)$$

Where $\theta_n(t) = 2\pi f_c\tau_n(t)$ is the phase of the n^{th} path.
The impulse response is,

$$h_b(t) = \sum_n \propto_n(t)e^{-j\theta_n(t)} \qquad (5)$$

The phase of each path can change by 2π radian when the delay $\tau_n(t)$ changes by $\frac{1}{f_c}$. If f_c is large, relative small motions in the medium can cause change of 2π radians.

Fading and Fading Channel Models

The wireless environment is highly unstable and fading is due to multipath propagation. Multipath propagation leads to rapid fluctuations of the phase and amplitude of the signal. The presence of reflectors in the environment surrounding a

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transmitter and receiver create multiple paths that a transmitted signal can traverse. As a result, the receiver sees the superposition of multiple copies of the transmitted signal, each traversing a different path. Each signal copy will experience differences in attenuation, delay and phase shift while traveling from the source to the receiver.

This can result in either constructive or destructive interference, amplifying or attenuating the signal power seen at the receiver. Fading may be large scale fading or small scale fading [9]. Based on multipath time delay spread small scale fading is classified as flat fading and frequency selective fading. If bandwidth of the signal is smaller than bandwidth of the channel and delay spread is smaller than relative symbol period then flat fading occurs whereas if bandwidth of the signal is greater than bandwidth of the channel and delay spread is greater than relative symbol period then frequency selective fading occurs. Based on Doppler spread small scale fading may be fast fading or slow fading. Slow fading occurs when the coherence time of the channel is larger relative to the delay constraint of the channel. The amplitude and phase change imposed by the channel can be considered roughly constant over the period of use. Slow fading can be caused by events such as shadowing, where a large obstruction such as a hill or large building comes in the main signal path between the transmitter and the receiver. Fast fading occurs when the coherence time of the channel is small relative to the delay constraint of the channel. The amplitude and phase change imposed by the channel varies considerably over the period of use. In a fast-fading channel, the transmitter may take advantage of the variations in the channel conditions using time diversity to help increase robustness of the communication.

Nakagami fading model considers the instance for multipath scattering with relatively large delay-time spreads, with different clusters of reflected waves. Within any one cluster, the phases of individual reflected waves are random, but the delay times are approximately equal for all waves. As a result the envelope of each cumulated cluster signal is Rayleigh distributed. The average time delay is assumed to differ significantly between clusters. If the delay times also significantly exceed the bit time of a digital link, the different clusters produce serious intersymbol interference, so the multipath self-interference then approximates the case of cochannel interference by multiple incoherent rayleigh-fading signals. Rayleigh fading model considers the fading is caused by multipath reception.

Rayleigh fading model assumes that the magnitude of a signal that has passed through transmission medium will vary randomly, or fade, according to a Rayleigh distribution. Rayleigh fading is a reasonable model when there are many objects in the environment that scatter the radio signal before it arrives at the receiver. Rayleigh fading is most applicable when there is no dominant line-of-sight propagation between the transmitter and receiver.

Rician model considers that the dominant wave can be a phasor sum of two or more dominant signals, e.g. the line-of-sight, plus a ground reflection. This combined signal is then mostly treated as a deterministic (fully predictable) process, and that the dominant wave can also be subject to shadow attenuation. This is a popular assumption in the modeling of satellite channels. Besides the dominant component, the mobile antenna receives a large number of reflected and scattered waves.

III. RICIAN MULTIPATH FADING CHANNEL

In this case of fading, the mobile receive, in addition to the other non LOS components, a direct signal from the source. The fading of the j^{th} path is a Rician distributed and can be modelized by:

$$R_j = \sqrt{X_{j_1}^2 + X_{j_2}^2}, \quad 1 \leq j \leq L, \quad (6)$$

Where, X_{j_1} (in-phase component) and X_{j_1} (quadrature component) are independent normally distributed RVs with the same variance σ^2 and means S_j cosγ_j and S_j sinγ_j, respectively, (γ_j is a random real number and S_j is the LOS amplitude of the jth path Rician fading). Thus, the normalized A_L is:

$$A_L = \frac{\sqrt{\sum_{j=1}^{L}\sum_{k=1}^{2} X_{jk}^2}}{\sqrt{\sum_{j=1}^{L} S_j^2 + 2L\sigma^2}} \quad (7)$$

Let's note $\beta = \left(\sum_{j=1}^{L}\left(\frac{S_j}{\sigma}\right)^2 + 2L\right)^{1/2}$. The RV$\beta A_L$ is a non-central chi distribution of 2L degrees of freedom, and non-centrality parameter,

$$\lambda = \left(\sum_{j=1}^{L}\left(\frac{S_j}{\sigma}\right)^2\right)^{1/2}$$

Its PDF is known to be [22]

$$f(r) = \lambda I_{L-1}(\lambda r)\left(\frac{r}{\lambda}\right)^L e^{-\frac{r^2+\lambda^2}{2}} \quad (8)$$

Where $I_{L-1}(.)$ is a modified Bessel function of the first kind of order $L-1$ [8].

Using the Jacobian transformation method, the PDF of A_L is given by:

$$f(r) = \beta\lambda I_{L-1}(\lambda\beta r)\left(\frac{\beta r}{\lambda}\right)^L e^{-\frac{(\beta r)^2+\lambda^2}{2}} \quad (8)$$

The Shannon capacity for the equivalent channel (3) is then,

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$$C = W\beta\lambda e^{-\frac{\lambda^2}{2}} \int_0^{+\infty} \log_2(1 + \bar{\gamma}r^2)\, I_{L-1}(\lambda\beta r) \left(\frac{\beta r}{\lambda}\right)^L e^{-\frac{(\beta r)^2}{2}} dr \quad (9)$$

Since the modified Bessel function of the first kind can be written as the infinite series [23],

$$I_{L-1}(\emptyset\sqrt{\gamma}) = \left(\frac{\emptyset\sqrt{\gamma}}{2}\right)^{L-1} \sum_{j=0}^{+\infty} \frac{1}{\Gamma(j+L)j!} \left(\frac{\emptyset\sqrt{\gamma}}{2}\right)^{2j} \quad (10)$$

The normalized Shannon capacity can also be written as:

$$\frac{C}{W} = \frac{e^{-\frac{\lambda^2}{2}}}{\ln(2)} \left(\frac{\beta^2}{2\bar{\gamma}}\right)^L \sum_{j=0}^{+\infty} \frac{1}{(j+L-1)j!} \left(\frac{\emptyset}{2}\right)^{2j} \times G_{2,3}^{3,1}\left[\frac{\beta^2}{2\bar{\gamma}} \middle| \begin{matrix} -j-L, 1-j-L \\ 0, -j-L, -j-L \end{matrix}\right] \quad (11)$$

This formula generalizes the capacity expression founded by Sagias et al. [6] in the case of one path. It generalizes also our first result corresponding to the normalized Rayleigh multipath fading channel. Indeed, for Rayleigh fading, all LOS amplitudes S_j equal zero $\left(\lambda = \phi = 0, \frac{\beta^2}{2} = L\right)$ and then we find the above expression.

IV. Nakagami-m Multipath Fading Channel

Let N be the Nakagami-m distributed RV of average energy $E[N^2] = 2\sigma^2$ and fading parameter,

$$m = \frac{(2a^2)^2}{E[(N^2 - 2a^2)^2]} \geq \frac{1}{2}$$

The PDF of N is given by [24]:

$$p(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{2\sigma^2}\right)^m r^{2m-1} e^{-\frac{mr^2}{2\sigma^2}} \quad (12)$$

The square of a Nakagami distribution $\Omega = N^2$ is a gamma distribution Γ(α,β) of parameters α = m and $\beta = \frac{2\sigma^2}{m}$ *and* PDF,

$$p(\gamma) = \frac{\left(\frac{m}{2\sigma^2}\right)^m}{\Gamma(m)} r^{m-1} e^{-\frac{m\gamma}{2\sigma^2}} \quad (13)$$

In the case of this fading, we show the expression of the channel capacity for two cases:

- The received average energies are equal.
  $$\sigma_i = \sigma, \forall_i \leq L$$
- The received average energies are not necessarily equal. This case generalizes the first one.

Case 1: received average energies are equal

Let $(|N_l|)_{1\leq l\leq L}$ be RVs Nakagami-m distributed of the same average energy $2\sigma^2$. Since the distribution of the two independent gamma RVs of parameters $(\alpha_1\beta)$ and $(\alpha_2\beta)$ is a gamma distribution of parameters $(\alpha_1 + \alpha_2\beta)$ [25], the RV $\sum_{l=1}^{L}|N_l|^2$ is a gamma distribution of parameters $\left(mL, \frac{2\sigma^2}{m}\right)$ and mean $2L\sigma^2$.

Furthermore, $\frac{1}{\sqrt{2L\sigma}}\sqrt{\sum_{l=1}^{L}|N_l|^2}$ is a normalized Nakagami-m distribution of fading parameter mL and PDF,

$$f(r) = \frac{2}{\Gamma(mL)} (mL)^{mL} r^{2mL-1} e^{-mLr^2} \quad (14)$$

It follows the expression of the normalized capacity of the equivalent channel model given by [6]:

$$C = W \frac{\left(\frac{mL}{\bar{\gamma}}\right)^{mL}}{\ln(2)\Gamma(mL)} G_{2,3}^{3,1}\left[\frac{mL}{\bar{\gamma}} \middle| \begin{matrix} -mL, 1-mL \\ 0, -mL, -mL \end{matrix}\right] \quad (15)$$

This expression generalizes again that of Sagias et al. [6]. Indeed, for m = 1, (15) reduces to the average capacity of the Rayleigh fading equivalent channel given by [6] and for L = 1 we got the channel capacity of the Nakagami fading channel (one path) shown in [6].

V. Conclusion

Carrying out literature review is very significant in any research project as it clearly establishes the need of the work and the background development. It generates related queries regarding improvements in the study already done and allows unsolved problems to emerge and thus clearly define all boundaries regarding the development of the research project. Plenty of literature has been reviewed for higher order statistics on different channels in communication system.

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