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# A Novel Hybrid Digital Watermarking using DWT, DCT and SVD

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Abstract— This paper aims at developing a hybrid image watermarking algorithm which satisfies both imperceptibility and robustness requirements. In order to achieve our objectives we have used singular values of Wavelet Transformation's sub bands to embed watermark. Further to increase and control the strength of the watermark, we use a scale factor. An optimal watermark embedding method is developed to achieve minimum watermarking distortion. An original image based recovery block's designed to securely embed the fragile watermarks so that the new method is robust to counterfeiting, even when the malicious attackers are fully aware of the watermark embedding algorithm. Experimental results are provided in terms of Peak signal to noise ratio (PSNR), Mean Squared Error (MSE), Weighted Peak signal to noise ratio (WPSNR) and correlation to demonstrate the effectiveness of the proposed algorithm. Results are taken by simulating the DWT, DCT and SVD watermarking techniques, in order make a good comparison model. Image operations such as JPEG compression from malicious image attacks: salt & pepper, Gaussian noise, sharpening, contrast enhancement, rotation and de-blurring.

# *Keywords*— Correlation, DCT, DWT, MSE, PSNR Watermarking, SVD, Wavelet transform.

#### I. INTRODUCTION

Digital media offers several distinct advantages over analog media, such as high quality, easy editing, high performance and easy duplication. The high spreading of broadband networks and new developments in digital technology has made ownership protection and authentication of digital multimedia a very important issue. Digital watermarking provides a possible solution to the problem of easy editing and duplication of images, since it makes possible to identify the author of an image by embedding secret information in it.

Introduction Digital information is easy to distribute, duplicate and modify which leads to the need for copyright protection techniques. Digital watermarking technique is one of the solutions to avoid unauthorized copying or tampering of multimedia data. Recently many watermarking schemes have been proposed to address this problem. Digital Watermarking is defined as the Ankit Jain Asst. Prof., Electronics and Communication Engineering Department Acropolis Institute of Technology & Research, Indore (M.P.), India ankitjain@acropolis.in

process of hiding a piece of digital data in the cover data which is to be protected and extracted later for ownership verification [1]. Some of the important applications of watermarking technique are copyright protection, ownership verification, finger printing, and broadcast monitoring. The features of watermarking include robustness and perceptibility. Robustness indicates the resistivity of watermark against different types of attacks such as cropping, rotating, scaling, low pass filtering, resizing, and addition of noise, JPEG compression, sharpness, histogram equalization and contrast adjustment. Those attacks are either intentional or unintentional. Robustness is the property which is important for ownership verification whereas the fragility is important for image authentication.

Robustness of watermarking algorithm is obtained to a maximum level when information is hidden in robust components of cover data. The increasing perceptibility will also decrease the quality of watermarked image. The watermarking schemes are broadly categories into two main domains i.e. spatial domain and the transform domain. In spatial domain watermarking the watermark is embedded by directly modifying the intensity values of the cover image. The most popular technique is the least significant bit (LSB) method. In transform domain the watermark is embedded by modifying the frequency coefficients of the transformed image.

The common methods in the transform domain are Fourier transform (DFT), discrete cosine transform (DCT), discrete wavelet transform (DWT), etc. Recently, singular value decomposition (SVD) was explored for watermarking. It is one of the most useful numerical analysis techniques having property that the singular values (SVs) of an image do not change significantly when a small perturbation is added to an image. [2-5]

It is well known that there are three main mutually conflicting properties of information hiding schemes: *capacity, robustness and indefectibility* [6]. It can be expected that there is no a single watermarking method or algorithm with the best quality in the sense that three mentioned above



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properties have the maximum value at once. But at the same time it is obvious that one can reach quite acceptable quality by means of combining various watermarking algorithms and by means of manipulations in the best way operations both in the spatial and in the frequency domains of an image. In paper [7] an approach to combining of DWT and DCT to improve the performance of the watermarking algorithms, which are based solely on the DWT, is proposed. Watermarking was done by embedding the watermark in the first and second level DWT sub-bands of the host image, followed by the application of DCT on the selected DWT sub bands. The combination of these two transforms performance watermarking improved the considerably when compared to the DWT-only watermarking approach. As a result this approach is at the same time resistant against copy attack. In addition, the fragile information is inserted in a way which preserves robustness and reliability of the robust part.

Robustness is the property which is important for ownership verification whereas the fragility is important for image authentication. Robustness of watermarking algorithm is obtained to a maximum level when information is hidden in robust components of cover data. The increasing perceptibility will also decrease the quality of watermarked image. Generally information could be hidden, directly by modifying the intensity value or pixel value of an image or its frequency components [8]. The former technique is called spatial domain technique and later is called frequency domain technique. To obtain frequency components of an image, it needs to be transformed using any one of the transformation techniques such as Discrete Fourier Transformation (DFT), Discrete short Fourier transformation (DSFT), Discrete Cosine Transformation (DCT) [9][10], Walsh Hadamard transformation (DHT) [11][12], and Discrete wavelet Transformation (DWT)[13][14][15][16]. In Transform domain casting of watermark can be done in full frequency band of an image or in specific frequency band such as in low frequency band or in high frequency band or in middle frequency band.

## II. LITERATURE REVIEW

In [14], two level decomposition of DWT is applied to transform an image into bands of different frequency and a particular band is converted into blocks of size 4x4 for embedding data. Each of those blocks is SVD transformed and watermark is hidden into diagonal matrix of every block. Extracted watermark from the attacked image is measured using the correlation factor NC. The algorithm shows that when DWT is combined with SVD technique the watermarking algorithm outperforms than the conventional DWT algorithm with respect to robustness against Gaussian noise, compression and cropping attacks.

In [15] the DWT is combined with SVD technique to hide data in high frequency band of an image. This scheme performs well for variety of image processing operations. Lahouari Ghouti, Ahmed Bouridane, Mohammad K. Ibrahim and Said Boussakta [17] have proposed a new perceptual model, which is only dependent on the image activity and is not dependent on the multifilter sets used. To achieve higher watermark robustness, the watermark embedding scheme is based on the principles of spread-spectrum communications.

Satisfying both imperceptibility and robustness for an image watermarking technique always remains a challenge because both are conflicting requirements. Since performing SVD on an image is computationally expensive, a hybrid DWT-SVDbased watermarking scheme is developed that requires less computation effort yielding better performance. Rather than embedding watermark directly into the wavelet coefficients, Chih-Chin Lai and Cheng-Chih Tsai have proposed to embed watermark in to the elements of singular values of the image's DWT sub bands. [18]

In order achieve both image authentication and protection simultaneously, Chun-Shien Lu, and

Hong-Yuan Mark Liao [19] proposes a cocktail watermarking which can resist different kinds of attacks and embed 2 watermarks (fragile & robust). Existing systems have used invariant properties of DCT coefficients and relationships between the coefficients for watermark embedding but they modify a large amount of data and produces maximum distortion. So a new method that uses Gaussian mixture model, Expectation Maximization algorithm, secret embedding key and private key for watermark embedding is proposed by Hua Yuan and Xiao- Ping Zhang [20].

Though there are existing systems that provides perceptual invisibility and robustness, YiweiWang, John F. Doherty & Robert E. Van Dyck [21] have proposed a new wavelet based technique for ownership verification by giving importance to the private control over the watermark and using randomly generated orthonormal filter banks. Liehua Xie and Gonzalo R.

Arce [22] have proposed a concept of using compression algorithms which are based on wavelet decompositions. In this approach, the SPIHT compression algorithm is executed to obtain a hierarchical list of the significant coefficients and at



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least 3 coefficients that correspond to the ones with the largest absolute is selected. The watermark is embedded into the host image based on the selected coefficients.

Mauro Barni, Franco Bartolini and Alessandro Piva [23] have proposed a new algorithm different from other existing systems in wavelet domain where the masking is performed pixel by pixel by taking into account the texture and the luminance content of all the image sub bands. A blind watermarking scheme that is robust against JPEG compression, Gaussian noise, salt and pepper noise, median filtering, and ConvFilter attacks was proposed by Ning Bi, Qiyu Sun, Daren Huang, Zhihua Yang, and Jiwu Huang [24]. This new approach uses Multiband wavelet transform and they embed the watermark bits in the mean trend of some middle-frequency sub images in the wavelet domain.

T. M. Ng and H. K. Garg [25] use a Laplacian model in place of Gaussian distribution along with the ML detection for better performance. Existing systems make use of wavelet coefficients and embed watermark bits directly into the coefficients whereas the system proposed by Shih-Hao

Wang and Yuan-Pei Lin [26] groups the wavelet coefficients into super tress and embed watermarks by quantizing super trees. Generally different resolutions of an image can be obtained using wavelet decomposition. Since human eyes are insensitive to the image singularities revealed by high frequency sub-bands, adding watermark to these singularities increases the quality of the image by providing imperceptibility. But the existing wavelets have limited ability to reveal singularities in all directions.

So Xinge You, Liang Du & Liang Du [27] construct the new nontensor product wavelet filter banks, which can capture the singularities in all directions. A novel multipurpose digital image watermarking method

[28] Has been proposed based on the multistage vector quantizer structure, which can be applied to image authentication and copyright protection applications.

#### III. PROPOSED METHODOLOGY Discrete Cosine Transform

A DCT is a Fourier related transform similar to Discrete Fourier Transform (DFT) but using only real numbers (since Fourier transform of real and even function is real and even). The DCT is a very popular transform function used in signal processing. It transforms a signal from spatial domain to frequency domain. Due to good performance, it has been used in JPEG standard for image compression. DCT has been applied in many fields such as data compression, pattern recognition, and image processing, and so on. Two dimensional discrete cosine transform (2D-DCT) is defined as:

F(jk)

$$= a(j)a(k) \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(mn) \cos\left[\frac{(2m+1)j\pi}{2N}\right] \cos\left[\frac{(2n+1)k\pi}{2N}\right]$$
(1)

The corresponding inverse transformation (IDCT) is defined as

$$= \sum_{m=0}^{f(mn)} \sum_{n=0}^{N-1} a(j)a(k)F(jk)\cos\left[\frac{(2m+1)j\pi}{2N}\right]\cos\left[\frac{(2n+1)k\pi}{2N}\right]$$
(2)

The DCT can not only concentrate the main information of original image into the smallest low frequency coefficient, but also it can cause the image blocking effect being the smallest, which can realize the good compromise between the information centralizing and the computing complication. So it obtains the wide spreading application in the compression coding.

DCT-based watermarking is based on two facts. The first fact is that most of the signal energy lies at low-frequencies sub band which contains the most important visual parts of the image. The second fact is that high frequency components of the image are usually removed through compression and noise attacks. The watermark is therefore embedded by modifying the coefficients of the middle frequency sub band so that the visibility of the image will not be affected and the watermark will not be removed by compression.

## Discrete Wavelet Transform (DWT)

The DWT is nothing but a system of filters. There are two filters involved, one is the "wavelet filter", and the other is the "scaling filter". The wavelet filter is a high pass filter, while the scaling filter is a low pass filter. DWT includes many kinds of transforms, such as Haar wavelet, Daubechies wavelet, and others. Figure 1 shows the workflow of DWT.



Figure 1: The workflow of discrete wavelet transform

After applying a 1-level DWT on an image, we get the approximation sub-band LL, the horizontal subband LH, the vertical sub-band HL, and the diagonal sub-band HH. Moreover, if we want to apply a 2level DWT on the image, we just simply apply another 1-level DWT on the approximation subband LL. After applying a 2-level DWT, we also get the approximation sub-band LL2, the horizontal sub-band LH2, the vertical sub-band HL2, and the diagonal sub-band HH2 of the approximation subband LL other than sub-bands LH, HL, HH.

# Watermark Embedding Process for DWT, DCT and SVD based Hybrid approach



Figure 2: Flow diagram of Embedding Process for DWT, DCT and SVD based Hybrid approach





Figure 3: Flow diagram of Extraction Process for DWT, DCT and SVD based Hybrid approach

## Singular value decomposition

Singular value decomposition is a linear algebra technique used to solve many mathematical problems [29]. The theoretical background of SVD technique in image processing applications to be noticed is [30]:

- a) The SVs (Singular Values) of an image has very good stability, which means that when a small value is added to an image, this does not affect the quality with great variation.
- b) SVD is able to efficiently represent the intrinsic algebraic properties of an image, where singular



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values correspond to the brightness of the image and singular vectors reflect geometry characteristics of the image.

c) An image matrix has many small singular values compared with the first singular value. Even ignoring these small singular values in the reconstruction of the image does not affect the quality of the reconstructed image.

#### **IV. SIMULATION & RESULTS**

In the evaluation of the performance of the watermarking scheme, we use the normalized mean square error MSE between the original and watermarked images, respectively, peak signal to noise ratio PSNR and Weighted peak signal to noise ratio WPSNR. The image pixels are assumed to be 8 bits to give a maximum pixel value of 255. The embedded watermarks cause imperceptible distortion at levels that provide reliable detection. The channel parameters like Peak signal to Noise ratio (PSNR), Weighted Peak signal to Noise ratio (WPSNR), Mean signal Error (MSE) and correlation have been calculated.

#### **PSNR**

PSNR is the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. Because many signals have a very wide dynamic range, PSNR is usually expressed in terms of the logarithmic decibel (dB) scale.

$$PSNR = 10 lg\left(\frac{255^2}{E}\right) dB \tag{3}$$

Where E is Mean Square Error, f(i, j) is pixel value of original image f'(i, j) of watermarked image and its logarithmic unit is dB given by formula:

$$E = \frac{1}{M \times N} \sum_{i=1}^{N} \sum_{j=1}^{M} [(f(i,j) - f'(i,j)]^2 \qquad (4)$$

#### **WPSNR**

The weighted PSNR (WPSNR) has been defined as an extension of the traditional PSNR. It weights each of the term of PSNR by local activity factor (linked to the local variance). The PSNR metric does not take into account image properties such as flat and textured regions. The watermark is embedding into textured regions and into edges, so the PSNR is inadequate to measure image quality in this case. The solution of this problem is using weighted PSNR.

$$WPSNR = 10 \log \frac{(L_{max^2})}{(MSE*NVF)^2}$$
(5)

Where,  $NVF = \frac{1}{1+\theta\sigma_x^2(i,j)}, \theta = \frac{D}{\sigma_{xmax}^2}$ Where  $\sigma_{xmax}^2$  is maximum local variance of a given image and  $D \in [50, 150]$  is a determined parameter.

#### *Correlation Coefficient (ρ)*

Comparability of extracted watermark with the original watermark is quantitatively analysed by using correlation coefficient. Value of  $\rho$  is between 0 and 1. The bigger the value of  $\rho$ , better is the robustness of watermark.

$$\rho(W, \bar{W}) = \frac{\sum_{i=1}^{r} W(i) \bar{W}(i)}{\sqrt{\sum_{i=1}^{r} \bar{W}^{2}(i)} \sqrt{\sum_{i=1}^{r} W^{2}(i)}}$$
(6)

Where W is the singular values of original watermark,  $\overline{W}$  is the extracted singular values and  $r = max(M_1, N_1)$ .

Following are the simulation results for the proposed DWT, DCT and SVD based Hybrid approach:



Figure 4: GUI for proposed work



Figure 5: Watermarked Image



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Figure 6: Recovered watermark



Figure 7: Salt and pepper attack image of watermarked image



Figure 8: Recovered watermark from salt and pepper



Figure 9: Gaussian noise attack image of watermarked image



Figure 10: Recovered watermark from Gaussian noise



Figure 11: Sharpen attack image of watermarked image



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Figure 12: Recovered watermark from Sharpen attack



Figure 13: Contrast enhancement of watermarked image



Figure 14: Rotation attack on watermarked image



Figure 15: De-blurring attack on watermarked image

| Table 1: DWT, | DCT and SVD image waterm | arking with and |
|---------------|--------------------------|-----------------|
|               | without attack           |                 |

| Attack   | PSNR   | MSE     | WPSN    | Correlatio |
|----------|--------|---------|---------|------------|
|          |        |         | R       | n          |
| Without  | 93.285 | 3.0518e | 121.559 | 0.99984    |
| Attack   | 3      | -005    | 2       |            |
| Salt &   | 50.169 | 0.62541 | 71.272  | 0.35666    |
| pepper   | 1      |         |         |            |
| Gaussian | 45.683 | 1.757   | 59.5965 | 0.20238    |
| noise    |        |         |         |            |
| Sharpene | 50.059 | 0.64141 | 68.7579 | 0.35955    |
| d        | 4      |         |         |            |
| Contrast | 52.350 | 0.37851 | 77.2433 | 0.4471     |
|          | 1      |         |         |            |
| Rotation | 47.346 | 1.1978  | 63.3483 | 0.25183    |
|          | 8      |         |         |            |
| De-      | 47.786 | 1.0824  | 64.4151 | 0.2669     |
| blurring | 9      |         |         |            |

#### V. CONCLUSION

The DWT technique provides better imperceptibility and higher robustness against attacks, at the cost of the DWT, DCT and SVD schemes. Each watermark bit is embedded in various frequency bands and the information of the watermark bit is spread throughout large spatial regions. As a result, the watermarking technique is robust to attacks in both frequency and time domains. Improvements in simulation performance can still be obtained by viewing the image watermarking problem. Our technique could also be applied to the multi resolution image structures with some modification about the choice of middle frequency coefficients. In this proposed method the values of the PSNRs of the watermarked images are always greater than 40 dB and it can effectively resist common image processing attacks, especially by JPEG compression and low-pass filtering.



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