BER Performance of Alamouti STBC and Precoded Alamouti in ZF and MMSE Equalization Techniques

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Abstract – Alamouti code is a simple space-time code that can be used for transmit diversity systems. This is a class of easily decoded space-time codes that achieve full diversity order in Rayleigh fading channels. Alamouti exist only for certain numbers of transmit antennas and do not provide array gain like diversity techniques that exploit transmit channel information. When channel state information (CSI) is available at the transmitter, though, precoding the space-time codeword can be used to support different numbers of transmit antennas. The objective of this paper is to design a method of precoders which shows comparison of bit error rate (BER) performance of Alamouti STBC and Precoded Alamouti in Zero Forcing and MMSE equalization techniques. The Rayleigh fading channel is used as modulation channel.

Keywords – Alamouti, BER, CSI, MMSE, Precoded Alamouti, Zero Forcing.

I. INTRODUCTION
In wireless channels, a signal sent from a transmitter does not follow a single path before it reaches at receiver. Instead, objects present in the environment cause it to traverse many different paths by means of physical effects such as reflection and refraction. Thus, multiple versions of the transmitted signal reach the receiver. The observed signal at the receiver is a sum of these multiple signals, and it is typically different from the originally transmitted one. Furthermore, in real applications, the relative positioning of transmitter-receiver pairs and the overall state of the objects between them may vary frequently in time, causing a change in the multiple links that signals follow. As a result, it is not rare that the signal observed by a receiver does not suffice to recover the actually transmitted signal. This factor, known as “multipath fading” or simply as “fading”, is a fundamental problem in wireless communication. Space-time coding was first described by Tarokh, Seshadri and Calderbank as a solution to this problem. It claims to increase the reliability of data transmission in wireless systems. Like many other wireless schemes, it is based on a technique known as diversity.

II. SPACE–TIME CODING
The explosion of wired and wireless telecommunication systems marked the end of the second millennium. The number of subscribers to the Internet or to a second-generation mobile network has increased exponentially. Hence, the users of both systems have augmented expectations in services and capacity. For wireless communication systems, a novel direction proposed to resolve capacity requirements in the challenging radio environment is the exploitation of Multiple Element Array (MEA) at both transmitter and receiver sides. Wireless systems with antenna elements at both edges are referred to in the literature as Multiple Input Multiple Output (MIMO) systems in contrast to Single Input Single Output (SISO) antenna systems that have one transmit and one receive antenna. Following this classification, a system with one transmit antenna and several receive antennas is denoted as Single Input Multiple Output (SIMO) system while several transmit antennas and one receive antennas designate a Multiple Input Single Output (MISO) system. In order to include the case where the transmitter and/or receiver antenna elements are not located in the same devices, MIMO systems are referred to as Multiple Transmitters Multiple Receivers (MTMR).

In this section, we introduce some general concepts about the signal processing used in MIMO systems, which is commonly called Space–Time Coding (STC).

Space–Time Block Codes (STBCs) are the simplest types of spatial temporal codes that exploit the diversity offered in systems with several transmit antennas. In 1998, Alamouti designed a simple transmission diversity technique for systems having two transmit antennas [1]. Alamouti’s scheme is very appealing in terms of implementation simplicity. Hence it motivates a search for similar schemes for more than two transmit antennas, to achieve diversity level higher than two. As a result, Orthogonal Space-Time Block Code (O-STBC) was introduced by Tarokh [2]. O-STBC is
generalizations of the Alamouti’s scheme to arbitrary number of transmit antennas. It retains the property of having linear maximum-likelihood decoding with full transmit diversity. In this research work, we analyze the different space time coding style for transmit diversity of MIMO system.

III. PRECODING
The benefits of using multiple antennas at both the transmitter and the receiver in a wireless system are well established. Multiple-input multiple-output (MIMO) systems enable a growth in transmission rate linear in the minimum number of antennas at either end. MIMO techniques also enhance link reliability and improve coverage. MIMO is now entering next generation cellular and wireless LAN products with the promise of widespread adoption in the near future.

While the benefits of MIMO are realizable when the receiver alone knows the communication channel, these can be further improved by transmitting the channel information at transmitter. The importance of transmit channel knowledge can be significant. For instance, in a four-transmit two-receive antenna system with independent identically distributed (I.I.D.) Rayleigh flat-fading, transmit channel knowledge can more than double the capacity at −5dB (SNR) signal-to-noise ratio and add 1.5 b/s/Hz additional capacity at 5 dB SNR. Such SNR choices are common in practical systems such as Wi-Fi and WiMax applications. In a non-I.I.D. channel, channel knowledge at the transmitter offers even greater leverage in performance. So, exploiting transmit channel side information is of great practical interest in MIMO wireless.

Precoding is a processing method that exploits CSIT by operating on the signal before transmission.

IV. PROPOSED METHODOLOGY
Figure 4.1 shows the basic block diagram for proposed system. Here rotational precoding is used which is explained as:

Precoding, when the codebook stored at both ends is obtained by quantizing rotational manifold is commonly referred to as rotational precoding. The main aim of rotational precoding is to direct all the power to the sub-streams along their corresponding Eigen directions of the channel, which can be achieved by the selection of appropriate codeword from the set that minimizes the distance

\[ d(W_k, W_l) = \frac{1}{\sqrt{2}} \left| \left| W_k W_k^H - W_l W_l^H \right| \right|_F \]  

Where \( d \) is the chordal distance.

Figure 2 shows MIMO system model with Precoder. \( M \) is the precoding matrix, \( Z \) is the equalizer, the stream \( \tilde{x} \) is processed, split into several sub-streams, pre-multiplied with codeword and transmitted. \( \tilde{y} \) is output.
The mathematical formulation is expressed as follows:
Consider the MIMO system with $N_T$ antennas, that is $h \in \mathbb{C}^{1 \times N_T}$. Let $C \in \mathbb{C}^{M \times T}$ denote a space-time codeword with a length of $M$, which is represented as:
$$C = [c_1, c_2, ..., c_T]$$
(2)
Where,
$$c_k = [c_{k,1}, c_{k,2}, ..., c_{k,M}]^T, \quad k = 1, 2, ..., T$$
and $M \leq N_T$ (3)
In the precoded STBC systems, the space-time codeword $C$ is multiplied by a precoding matrix $W \in \mathbb{C}^{N_T \times M}$, which is chosen from the codebook.
$$F = \{W_1, W_2, W_3, ..., W_L\}$$
(4)
The objective is to choose an appropriate codeword that improves the overall system performance such as channel capacity or error performance. Assuming that $N_T$ channels remain static over $T$, the received signal $y \in \mathbb{C}^{1 \times T}$ can be expressed as,
$$y = \sqrt{\frac{E}{N_T}} h W C + z$$
(5)
In above equation the length of each vector is $M \leq N_T$. The probability of codeword error can be derived as follows: For a given channel $h$ and precoding matrix $W$, we consider the pair wise codeword error probability $P_r(C_i \rightarrow C_j|H)$. The upper bound of the pair wise error probability is given as:
$$P_r(C_i \rightarrow C_j|H) = Q\left(\frac{\sqrt{\rho \|H W E_i j\|^2}}{2N_T}\right)$$
$$\leq \exp\left(-\frac{\rho \|H W E_i j\|^2}{4N_T}\right)$$
(6)
Where $\rho$ is the signal-to-noise ratio (SNR), given as $\rho = E_s/N_0$ and $E_{i,j}$ is the error matrix between the codewords $C_i$ and $C_j$ which is defined as $E_{i,j} = C_i - C_j$ for a given STBC scheme. From equation above we see that $\|H W E_i j\|^2_\bar{F}$ needs to be maximized in order to minimize the pairwise error probability. This leads us to the following codeword selection criterion:
$$W_{opt} = \arg \max_{W \in F, i \neq j} \|H W E_{i,j}\|^2_\bar{F}$$
$$= \arg \max_{W \in F, i \neq j} \text{Tr}(H W E_{i,j} E_{i,j}^H W^H H^H)$$
$$= \arg \max_{W \in F} \text{Tr}(H W H^H)$$
(7)
In the course of deriving equation (7), we have used the fact that the error matrix of STBC has the property of \( E_{i,j}E_{i,j}^H = aI \) with constant \( a \). When the constraint \( W \in F \) is not imposed, the above optimum solution \( W_{opt} \) is not unique, because \( ||HW_{opt}||_F^2 = ||HW_{opt}Z||_F^2 \). Where \( Z \) is a unitary matrix. The unconstrained optimum solution of equation (7) can be obtained by singular value decomposition (SVD) of channel \( H = USV^H \), where the diagonal entry of \( S \) is in descending order. It is shown that the optimum solution of above equation is given by the leftmost \( M \) columns of \( V \), that is,

\[
W_{opt} = [v_1 | v_2 | \ldots | v_M] \Delta \tilde{V}
\]

Since \( \tilde{V} \) is unitary, \( \lambda_i(W_{opt}) = 1, i = 1,2, \ldots, M \) where \( \lambda_i(A) \) denotes the \( i \)th largest eigenvalue of the matrix \( A \).

In case that a channel is not deterministic, the following criterion is used for the codebook design:

\[
E \left( \min_{W \in F} \left( ||HW_{opt}||^2 - ||HW||^2 \right) \right) \leq E \left( \lambda_i^2(H) E \left( \min_{W \in F} \frac{1}{2} ||\tilde{V}V^H - WW^H||^2 \right) \right)
\]

Since \( \lambda_i^2(H) \) is given, the codebook must be designed so as to minimize,

\[
E \left( \min_{W \in F} \frac{1}{2} ||\tilde{V}V^H - WW^H||^2 \right) \]

in equation (10).

V. SIMULATION AND RESULTS

The performance of proposed technique has been studied by means of MATLAB simulation.

Figure 3 shows the performance of Alamouti STBC and Precoded Alamouti STBC system with Zero Forcing equalization technique. At SNR=15dB, BER performance of Precoded Alamouti is \( 10^{-2.9} \) while without precoding it is \( 10^{-1.9} \). It is clear that precoded STBC performs better than normal STBC system.

VI. CONCLUSION

The most prominent space-time block codes (STBCs) is the Alamouti code. The approach in this dissertation is to isolate the analysis of precoded MIMO systems. The proposed precoded Alamouti have a layered structure, which is implemented in the simplest Grassmannian precoding. Simulation results show comparison of BER performance between the Alamouti STBC and Precoded Alamouti for Zero Forcing and MMSE equalization techniques. And finally it is found that the Precoded Alamouti shows better results in terms of BER as compared to other schemes for Z-F and MMSE techniques.

REFERENCE


