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Comparative Analysis of MUSIC Algorithm in Smart Antenna

Tithi Gunjan
M. Tech. Scholar, Department of Electronics and Communication Engineering
Acropolis Institute of Technology and Research, Indore (M.P.), India
tithi.gunjan@gmail.com

Gaurav Chaitanya
Department of Electronics and Communication Engineering
Acropolis Institute of Technology and Research, Indore (M.P.), India

Abstract – **High-resolution signal parameter estimation is a major problem in many signal processing applications. Such applications contain direction of arrival (DOA) estimation for narrow band signals and wideband signal emitted by multiple sources and received by sensor arrays. It is well known that MUSIC algorithm outperforms any other method existing in the literature. This paper uses different MUSIC algorithms for DOA estimation viz; Simple MUSIC Algorithm, ROOT-MUSIC Algorithm, Spatially Smoothed MUSIC Algorithm and Toeplitz approximation based MUSIC algorithm. The matrix Toeplitz approximation recovers the Toeplitz structure, and the approximated matrix is used to obtain more accuracy estimates via MUSIC method.**

Keywords – **DOA, MUSIC, ROOT MUSIC, Toeplitz Matris.**

I. INTRODUCTION

Conventional base station antennas are usually omnidirectional. It allies some sort of power loss because most of it will be transmitted other directions than toward the desired individual. Furthermore, additional end users will feel the power radiated in other directions as interference. It causes spectral efficiency degradation.

A promising technique to increase the spectrum efficiency is using smart antennas. An smart antenna is a multi element antenna in which the signals received at each element are intelligently and adaptively combined to improve the overall performance of the wireless system with the reverse performance on transmit [1].

The smart antenna technology is based on antenna arrays where the radiation pattern is altered by adjusting the amplitude and relative phase on the different elements [2]. Smart antennas have the property of spatial filtering, which make it possible to receive energy from a particular direction while simultaneously blocking it from another direction. The benefit of smart antenna is that they can increase range and capacity of system while helping to eliminate both interference and fading.

II. DIRECTION OF ARRIVAL ESTIMATION

Direction of arrival (DOA) estimation plays an important role for smart antenna system at the receiver end. The purpose of DOA estimation is to use the data received by the array to estimate the direction of arrival of the signal. DOA determines the angle of arrival of the incoming signals. The successful design of adaptive array smart antenna depends highly on the performance of DOA estimation algorithm. Thus, the smart antennas system becomes capable to locate and track signals by the both: users and interferers.

In the modelling of adaptive array smart antenna for mobile communication the performance of DOA estimation algorithm depends on several parameters such as number of mobile users and their space distribution, the number of array elements and their spacing, the number of signal samples and SNR.

Number of DOA estimation algorithms have been developed and categorized into two methods viz.

- Conventional type
- Subspace type

Conventional methods also called classical methods which first compute a spatial spectrum and then estimate DOAs by local maxima of the spectrum. Methods that fall under this class are Bartlett, Capon methods. But these methods suffer from lack of angular resolution. For this reason high angular resolution subspace methods such as MUSIC and ESPRIT algorithms are most used; they are more accurate and not limited to physical size of array aperture. MUSIC algorithm is highly accurate and stable and provides high angular resolution compared to ESPRIT and hence MUSIC algorithm can be widely used in mobile communication to estimate the DOA of the arriving signals.

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III. MUSIC ALGORITHM

MUltiple SIgnal Classification (MUSIC) is the most popular technique used in Direction of arrival estimation. It is high resolution technique based on exploiting the eigenstructure of input covariance matrix. MUSIC makes assumption that the noise in each channel is uncorrelated making correlation matrix diagonal. The incident signals are somewhat correlated creating non diagonal signal correlation matrix.

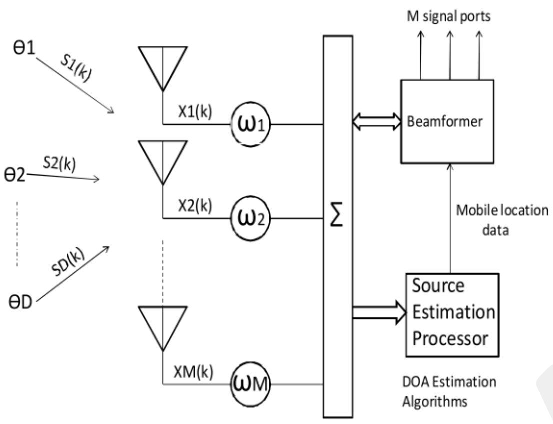

Figure 1: M element antenna array with D arriving signals

Mathematical model for MUSIC algorithm

If the number of signals impinging on M element array is D, the number of signal eigenvalues and eigenvectors is D and number of noise eigenvalues and eigenvectors is M-D. The array correlation matrix with uncorrelated noise and equal variances is then given by:

$$R_{XX} = A * R_{SS} * A^H + \sigma_n^2 * I \quad (1)$$

where $A = [a(\theta_1)a(\theta_2)a(\theta_3) \dots a(\theta_D)]$ is M × D array steering matrix.

$R_{SS} = [s_1(k)s_2(k)s_3(k) \dots \dots s_D(k)]^T$ is D × D source correlation matrix.

R_{XX} has D eigenvectors associated with signals and M – D eigenvectors associated with the noise. We can then construct the M×(M-D) subspace spanned by the noise eigenvectors such that

$$V_N = [V_1 V_2 V_3 \dots V_{M-D}] \quad (2)$$

The noise subspace eigenvectors are orthogonal to array steering vectors at the angles of arrivals θ_1, θ_2, θ_3, θ_D and the MUSIC Pseudospectrum is given as:

$$P_{MUSIC(\theta)} = 1/abs((a(\theta))^H * V_N * V_N^{H*} a(\theta) \quad (3)$$

However when signal sources are coherent or noise variances vary the resolution of MUSIC diminishes. To overcome this we must collect several time samples of received signal plus noise, assume ergodicity and estimate the correlation matrices via time averaging as:

$$R_{XX} = \frac{1}{K}\sum_{k=1}^{K} X(k) * X(k)^H \quad (4)$$

And

$$R_{XX} = A * R_{SS} * A^H + A * R_{sn} + R_{ns} * A^H + R_{nn} \quad (5)$$

The MUSIC Pseudospectrum using equation no. (3) with time averages now provides high angular resolution for coherent signals.

ROOT-MUSIC Algorithm

For the case of a uniformly spaced linear array with interelement spacing d, the m^th element of the steering vector $a(\emptyset)$ may be expressed as:

$$a_m(\emptyset) = exp\left(j2\pi m\left(\frac{d}{\lambda}\right)\cos(\emptyset)\right)$$
$$m = 1, \dots, M \quad (6)$$

The MUSIC spectrum is an all-pole function of the form:

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\emptyset)V_n V_n^H a(\emptyset)} = \frac{1}{a^H(\emptyset)Ca(\emptyset)} \quad (7)$$

Where, $C = V_n V_n^H$. Using equation (7) may be written as:

$$P_{MUSIC}^{-1} = \sum_{m=1}^{M}\sum_{n=1}^{M} \exp\left(-j\frac{2\pi md}{\lambda}\cos\emptyset\right) C_{mn} \exp\left(j\frac{2\pi md}{\lambda}\cos\emptyset\right) \quad (8)$$

Where C_{mn} is the entry in the m^th row and n^th column of C. Combining the two summations into one, (8) can be simplified as:

$$P_{MUSIC}^{-1} = \sum_{n=1}^{M} C_l \exp\left(j\frac{2\pi d}{\lambda} l \cos\emptyset\right) \quad (9)$$

Where $C_l = \sum_{m-n=l} C_{mn}$ is the sum of the entries of C along the i^th diagonal. By defining a polynomial $D(Z)$ as follows,

$$D(Z) = \sum_{l=-M+1}^{M+1} C_l Z^{-1} \quad (10)$$

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Evaluating the MUSIC spectrum $P_{MUSIC}(\theta)$ becomes equivalent to the polynomial $D(Z)$ on the unit circle, and the peaks in the MUSIC Spectrum exist because the roots o $D(Z)$ flie close to the unit circle. Ideally, with no noise, the poles will lie in exactly on the unit circle at locations determined by the DOA. In other words, a pole of $D(Z)$ at,

$z = z_1 = |z_1|\exp(j\ \arg(z_1))$ will result in a peak in the MUSIC spectrum at $\cos\emptyset = \left(\frac{\lambda}{2\pi d}\right)\arg(z_1)$.

Spatially Smoothed Music Algorithm

The signal covariance matrix R_{SS} is a full-rank matrix (i.e., non-singular) as long as the incident signals on the sensor array are uncorrelated, which is the key to the MUSIC eigenvalues decomposition. However, if the incident signals become highly correlated, which is a realistic assumption in practical radio environments, matrix R_{SS} will lose its non-singularity property and, consequently, the performance of MUSIC will degrade severely. In this case, spatial smoothing (SS) must be used to remove the correlation between the incident signals by dividing the main sensor array into forward/backward overlapping subarrays and introducing phase shifts between these sub-arrays.

The vector of the received signals at the k^{th} forward subarray is expressed as:

$$x_k^F(t) = AD^{(k-1)}s(t) + n_k(t) \qquad (11)$$

Where $(k-1)$ denotes the k^{th} power of the diagonal matrix D given by:

$$D = diag\left\{e^{-j\frac{2\pi}{\lambda}\sin\theta_1}, \dots, e^{-j\frac{2\pi}{\lambda}\sin\theta_m}\right\} \qquad (12)$$

The spatial correlation matrix R of the sensor array is then defined as the sample mean of the covariance matrices of the forward sub-arrays:

$$R = \frac{1}{L}\sum_{k=0}^{L-1} R_k^F \qquad (13)$$

Here L is the number of overlapping sub-arrays. When applying FSS, the N-elements sensor array can detect up to N/2 correlated signals [3].

Toeplitz approximation method

In order to improve the estimation performance, Toeplitz approximation is introduced. The S. Y. Kung et al. [4] proposed a Toeplitz Approximation Method (TAM) which is based on a reduced order Toeplitz approximation of an estimated spatial covariance matrix. The estimated covariance matrix, in the case in which sources are uncorrelated and statistically stationary, is Toeplitz. In a multipath environment, however, the source paths are fully correlated, and this covariance matrix is not Toeplitz. The Toeplitz structure can he guaranteed by employing spatial smoothing, which destroys cross correlation between directional components. The TAM is designed for robustness in an arbitrary ambient noise environment.

IV. Simulation and Results

Simulation is carried out using MATLAB 2010a:

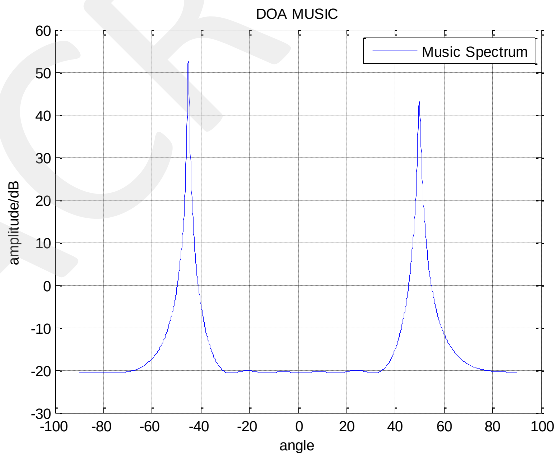

Figure 2: MUSIC pseudo spectrum found with MUSIC for $\theta_1 = -45^\circ$ and $\theta_2 = 50^\circ$

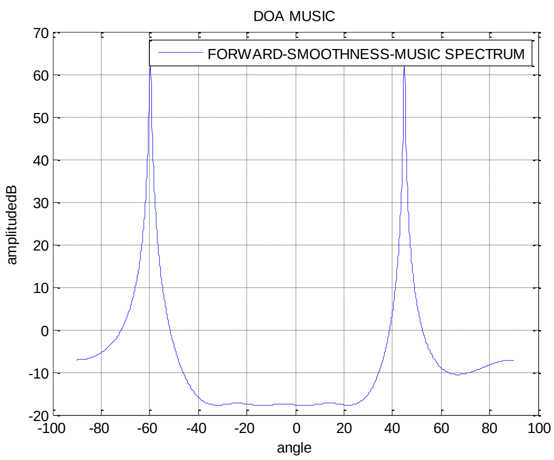

Figure 3: MUSIC pseudospectrum found with Forward MUSIC for $\theta_1 = -60^\circ$ and $\theta_2 = 45^\circ$

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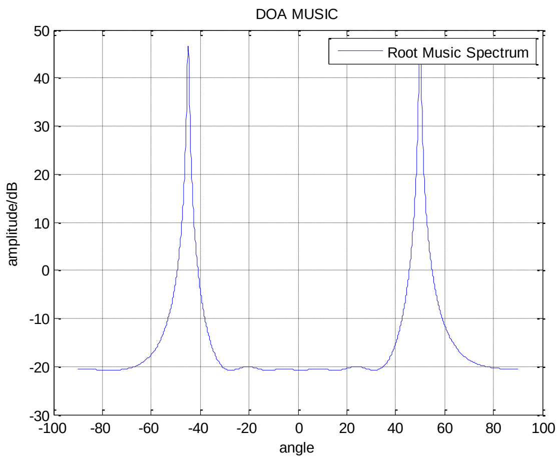

Figure 4: MUSIC pseudo spectrum and roots found with root-MUSIC for $\theta_1 = -45^\circ$ and $\theta_2 = 50^\circ$

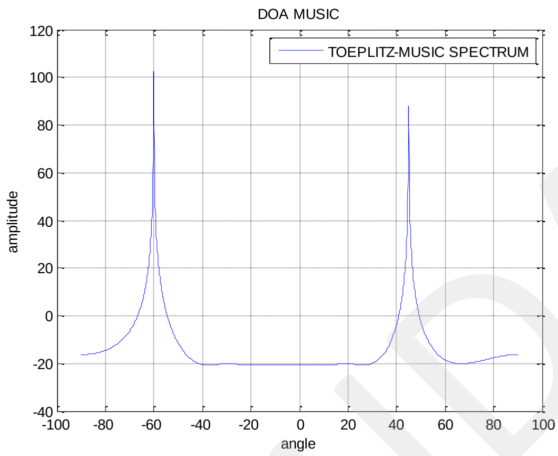

Figure 5: MUSIC pseudo spectrum found with Toeplitz-MUSIC for $\theta_1 = -45^\circ$ and $\theta_2 = 50^\circ$

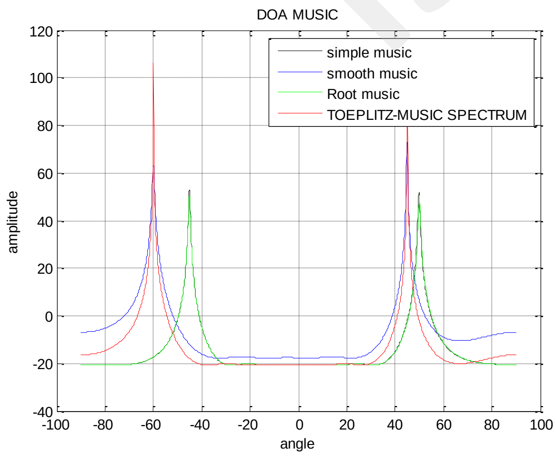

Figure 6: Comparative MUSIC pseudo spectrum for different algorithms

V. CONCLUSION

The MUSIC algorithm has greater resolution and accuracy than the other algorithms (i.e. Bartlett, CAPON) and hence they are being investigated much in detail in much literature. The results show the performance of simple MUSIC, root MUSIC and toeplitz music is better that smooth music. The performance can be improved with more elements in the array, with higher number of snapshots of signals and greater angular separation between the signals. These are responsible for the form of sharper peaks in MUSIC spectrum and smaller errors in angle detection.

REFERENCES

[1] R. K. Jain, Sumit Katiyar and N. K. Agrawal, “Smart Antenna for Cellular Mobile Communication” VSRD International Journal of Electrical, Vol. 1, No. 9, PP. 530-541, 2011.

[2] T. B. Lavate, V. K. Kokate & A. M. Sapkal, “Performance Analysis of MUSIC and ESPRIT DOA Estimation Algorithms for Adaptive Array Smart Antenna in Mobile Communication”, International Journal of Computer Netwoks (IJCN), Vol. 2, Issue 3, PP. 152-158, 2010.

[3] E. M. Al-Ardi, R. M. Shubair, and M. E. Al-Mulla, “Computationally Efficient DOA Estimation in a Multipath Environment Using Covariance Differencing and Iterative Spatial Smoothing,” IEEE, 2005.

[4] Kung S Y, Lo C K, Foka R. “A toeplitz approximation approach to coherent source direction finding,” IEEE international conference ICASSP-86, vol. 11, pp. 193-196, 1986.

[5] Qing Wang, Hua Chen, Guohuang Zhao, Bin Chen and Pichao Wang, “An Improved Direction Finding Algorithm Based on Toeplitz Approximation”, Sensors, ISSN: 1424-8220, 2013.

[6] Porat B., Friedlander B., “Direction finding algorithms based on high-order statistics”, IEEE Trans. Signal Process. 1991, 39, 2016–2024.

[7] J. G. Arceo-Olague, D. H. Covarrubias-Rosales, J. M. Luna-Rivera, A. Ángeles-Valencia, “Efficient Adaptive Algorithms for DOA Estimation in Wireless Communications”, Internation Journal of Communications Network and System Sciences, 173-176, 2010.