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BER Analysis of Precoders in Multi-User MIMO

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Abstract – **For Multi-Input Multi-Output (MIMO) transmission systems, we present a diagonal Precoder with a Minimum Bit Error Rate (MBER). This research builds on the findings with optimized the global transmission system (precoder and equalizer) using the Minimum Mean Square Error (MMSE) criterion, a new diagonal precoder that minimizes the BER is used to optimize the system. Our research is inspired by the notion that people are more likely to favor a solution that reduces the BER over the Mean Square Error from a practical standpoint. Monte Carlo simulations employing a Quadratic Amplitude Modulation are used to demonstrate the performance improvement (QAM).**

Keyword – **MBER, MIMO, MMSE, Precoder, QAM, etc.**

I. INTRODUCTION

The data transmitted by dividing with more than one antenna on the transmitter side is received by more than one antenna on the receiver side in MIMO systems, which is one of the multiple antenna technologies. These obtained data are merged, boosting the connection's reliability. The Space Time Block Codes (STBC), which are part of the Space Time Coding family, are one of the approaches that create diversity in the transmitter. The Alamouti approach, which is the simplest and most efficient variant of this, produces the most diversity increase by using the maximum similarity (Maximum Likelihood, ML) method [1]. Recent research on STBC and associated MIMO signal processing has gotten a lot of interest. One of the most important approaches in wireless communications is coded modulation. The difficulties and aims of wireless communication systems are always high spectral efficiency and link reliability. Multiple-input multiple-output (MIMO) technology using bit-interleaved coded modulation (BICM) [2] has become one of the essential technologies in recent wireless communication standards, such as IEEE 802.11ac and 3GPP LTE [3-5].

Spatial multiplexing is a good way to increase channel capacity and spectral efficiency in a MIMO system. Each low-rate substream is broadcast simultaneously while a high-data-rate signal stream is mapped onto numerous layers [6] [7]. Unfortunately, poor channel matrix conditioning makes spatial multiplexing vulnerable. In addition, MIMO can be used to increase diversity gain. The most widely utilized technologies are precoding and space-time coding (STC). Precoding is based on the use of channel state information (CSI) at the transmitter and receiver to tailor the transmitted signal to the matrix channel's Eigen structure [8] [9] [10]. MIMO channel capacity is known to be achieved via precoding based on singular value decomposition (SVD) with complete CSI. The transmit precoding and receiver shaping change the MIMO channel into M independent single-input single-output (SISO) channels after the channel matrix SVD [11]. The requirement of perfect CSI at both the transmitter and receiver is the most important disadvantage of SVD precoding. Limited feedback (LF) precoding methods were proposed to reduce the data for the feedback channel [12] [13] [14], where both the transmitter and the receiver are aware of a finite set of pre-determined unitary precoding matrices, referred to as the unitary codebook. Over a limited feedback channel, the receiver just needs to feedback the index of the precoding matrix as a function of the current CSI. MIMO systems' feedback overhead is greatly reduced with this practical solution [15] [16].

II. PROPOSED SYSTEM MODEL

A. Rotational Precoding

Precoding, when the codebook stored at both ends is obtained by quantizing rotational manifold is commonly referred to as rotational precoding. The main aim of rotational precoding is to direct all the power to the sub-streams along their corresponding Eigen directions of the channel, which can be achieved by the selection of appropriate codeword from the set that minimizes the distance

$$d(W_k, W_l) = \frac{1}{\sqrt{2}} ||W_k W_k^H - W_l W_l^H||_F \quad (1)$$

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Where d is the chordal distance.

B. Precoded STBC

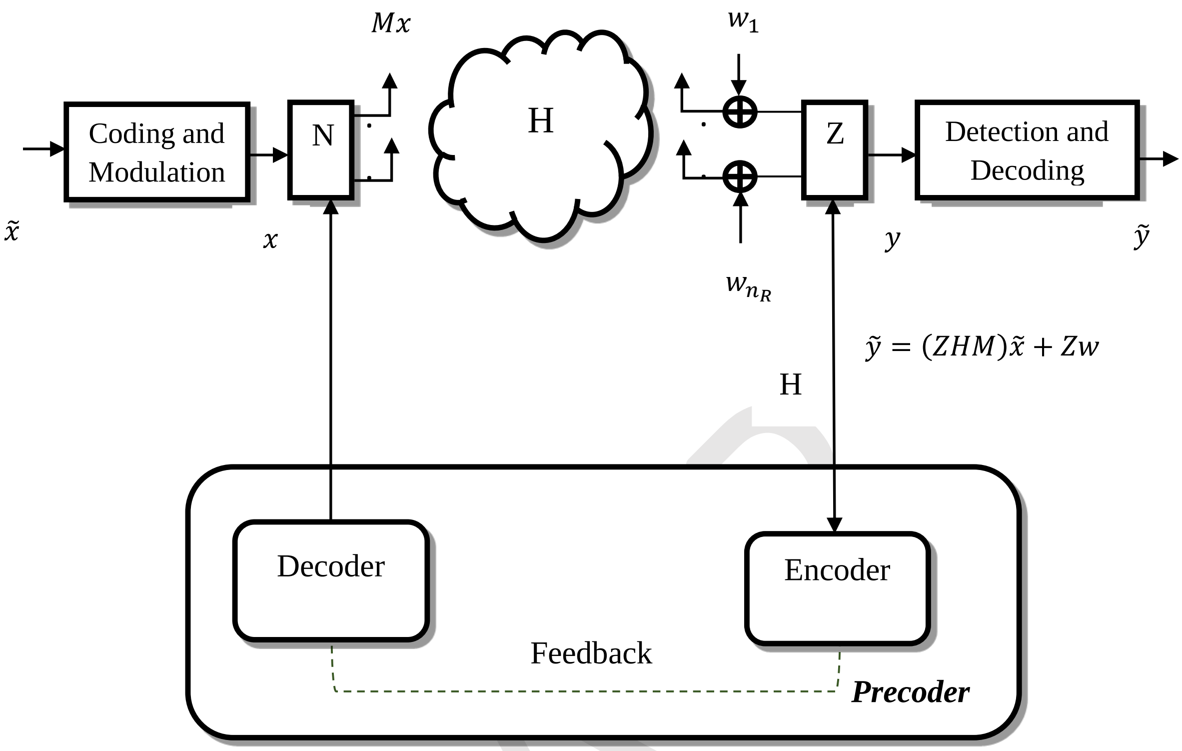

Figure 1: MIMO system model with Precoder

Figure 1 shows MIMO system model with Precoder. M is the precoding matrix, Z is the equalizer, the stream $\tilde{x}$ is processed, split into several sub-streams, pre-multiplied with codeword and transmitted. $\tilde{y}$ is output.

The mathematical formulation is expressed as follows:

Consider the MIMO system with N_T antennas, that is $h \in C^{1\times N_T}$. Let $C \in C^{M\times T}$ denote a space-time codeword with a length of M, which is represented as:

$$C = [c_1 c_2 \dots \dots c_T] \tag{2}$$

Where,

$$c_k = [c_{k,1} c_{k,2} \dots \dots c_{k,M}]^T, \qquad k = 1,2, \dots \dots, T \text{ and } M \le N_T \tag{3}$$

In the precoded STBC systems, the space-time codeword C is multiplied by a precoding matrix $W \in C^{N_T \times M}$, which is chosen from the codebook.

$$F = \{W_1, W_2, W_3 \dots, W_L\} \tag{4}$$

The objective is to choose an appropriate codeword that improves the overall system performance such as channel capacity or error performance. Assuming that N_T channels remain static over T, the received signal $y \in C^{1\times T}$ can be expressed as:

$$y = \sqrt{\frac{E_X}{N_T}} hWC + z \tag{5}$$

In above equation the length of each vector is $M \le N_T$. The probability of codeword error can be derived as follows: For a given channel h and precoding matrix W, we consider the pair wise codeword error probability $P_r(C_i \to C_j|H)$. The upper bound of the pair wise error probability is given as:

$$P_r(C_i \to C_j|H) = Q\left(\sqrt{\frac{\rho \left|\left|HWE_{i,j}\right|\right|_F^2}{2N_T}}\right) \le exp\left(-\frac{\rho \left|\left|HWE_{i,j}\right|\right|_F^2}{4N_T}\right) \tag{6}$$

Where ρ is the signal-to-noise ratio (SNR), given as $\rho = E_x/N_0$ and $E_{i,j}$ is the error matrix between the

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codewords C_i and C_j which is defined as $E_{i,j} = C_i - C_j$ for a given STBC scheme.

When the constraint $W \epsilon F$ is not imposed, the above optimum solution W_{opt} is not unique, because $||HW_{opt}||_F^2 = ||HW_{opt}Z||_F^2$. Where Z is a unitary matrix. The unconstrained optimum solution of equation (6) can be obtained by singular value decomposition (SVD) of channel $H = U\Sigma V^H$, where the diagonal entry of S is in descending order. It is shown that the optimum solution of above equation is given by the leftmost M columns of V, i.e.,

$$W_{opt} = [v_1 v_1 \ldots\ldots v_M] \triangleq \bar{V} \quad (7)$$

Since $\bar{V}$ is unitary, $\lambda_i(W_{opt}) = 1, i = 1,2,\ldots,M$ where $\lambda_i\,(A)$ denotes the i^{th} largest eigenvalue of the matrix A.

In case that a channel is not deterministic, the following criterion is used for the codebook design:

$$E\left\{\underbrace{min}_{W\in F}\left(\left||HW_{opt}|\right|_F^2 - \left||HW|\right|_F^2\right)\right\} \quad (8)$$

Where the expectation is with regards to the random channel H. W_{opt} in equation (8) follows from equation (7) for the given channel H.

The above expected value in equation (8) is upper-bounded as:

$$E\left\{\underbrace{min}_{W\in F}\left(\left||HW_{opt}|\right|_F^2 - \left||HW|\right|_F^2\right)\right\} \leq E\{\lambda_1^2\{H\}\}E\left\{\underbrace{min}_{W\in F}\frac{1}{2}||\bar{V}\bar{V}^H - WW^H||_F^2\right\} \quad (9)$$

Since $\lambda_1^2\{H\}$ is given, the codebook must be designed so as to minimize,

$E\left\{\underbrace{min}_{W\in F}\frac{1}{2}||\bar{V}\bar{V}^H - WW^H||_F^2\right\}$ in equation (9).

C. Equalization Techniques

1. Zero Forcing Equalizer

Zero Forcing Equalizer otherwise stated as ZF equalizer in this research; falls under the category of linear equalizers. ZF equalizer reverses the frequency response of the channel. The name Zero forcing is because this equalizer brings ISI to zero level in an ideal noise free case. ZF equalizer is useful in situations where ISI is more dominant than noise.

Let H denote the channel matrix and x be the transmitted signal vector, then ZF equalizer can be implemented by multiplication of inverse channel matrix with the received signal to produce the estimate of transmitted signal $\tilde{x}$:

$$\tilde{x} = H^{\dagger} r = H^{\dagger}(Hx) = x \quad (10)$$

Where $(.)^{\dagger}$ denotes the pseudo-inverse.

When noise is also considered then the resulting signal will be:

$$\tilde{x} = H^{\dagger}R = H^{\dagger}(Hx+n) = x + H^{\dagger}n \quad (11)$$

It is clear that, the estimate signal $\tilde{x}$ from zero forcing equalizer is a decoded signal with addition to inverted channel matrix and unknown noise. Due to this noise amplification property of ZF equalizer, MMSE equalizer was proposed.

2. MMSE Equalizer

Minimum Mean Square Error equalizer otherwise stated as MMSE equalizer throughout this research reduces the problem of noise amplification by taking Noise power into consideration while designing filtering matrix through MMSE criterion. The estimated symbol vector from MMSE equalizer can be given as:

$$\tilde{x} = [[(H^H H + (\sigma^2 I))^{-1}]\, H^H]\, r \quad (12)$$

Where H represents H channel matrix, σ^2 is noise variance.

III. Simulation Results

Table 1: Comparison of BER for Alamouti MMSE and Precoded Alamouti MMSE with QPSK for different SNR values

SNR	BER for Alamouti MMSE with QPSK	BER for Precoded Alamouti MMSE with QPSK
0	0.4177	0.0050
3	0.2919	0.0034
6	0.1802	0.0021
9	0.1038	0.0012
12	0.0558	0.0007
15	0.0302	0.0004
18	0.0144	0.0002
21	0.0094	0.0001

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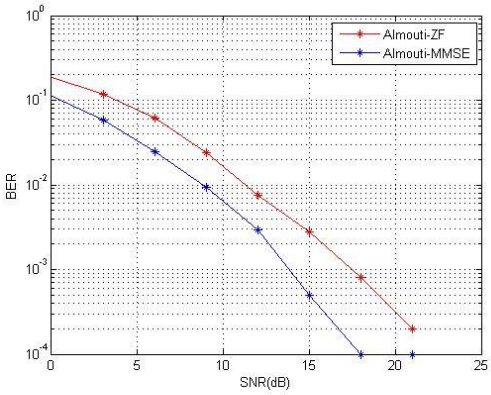

Figure 2: Comparison of BER performance of Alamouti Scheme for BPSK modulation without precoding in ZF and MMSE equalization

Table 2: Comparison of BER for Alamouti ZF and Alamouti MMSE with BPSK for different SNR values

SNR	BER for Alamouti-ZF with BPSK	BER for Alamouti-MMSE with BPSK
0	0.2125	0.0209
3	0.1483	0.0147
6	0.0897	0.0095
9	0.0490	0.0051
12	0.0302	0.0031
15	0.0132	0.0015
18	0.0088	0.0008
21	0.0040	0.0004

Figure 2 shows the performance of Alamouti STBC for BPSK modulation without precoding in Zero Forcing and MMSE equalization techniques. BER performance of Alamouti-ZF is 4×10^{-3} and for Alamouti-MMSE, it is 4×10^{-4} at 21 db. Hence, it is clear that the Alamouti-MMSE performs better than Alamouti-ZF system.

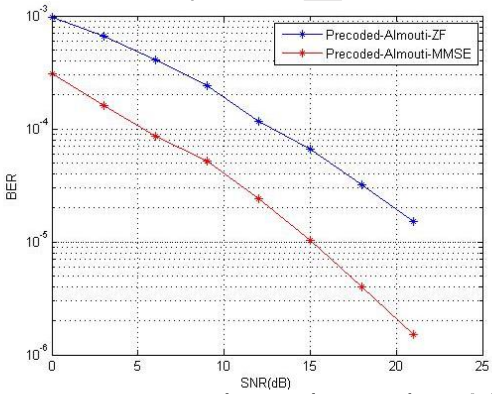

Figure 3: Comparison of BER performance of Precoded Alamouti Scheme for BPSK modulation in ZF and MMSE equalization

Table 3: Comparison of BER for precoded Alamouti ZF and Precoded Alamouti MMSE with BPSK for different SNR values

SNR	BER for Precoded Alamouti-ZF with BPSK	BER for Precoded Alamouti-MMSE with BPSK
0	0.0019	0.0006450
3	0.0013	0.0003380
6	0.0008	0.0001860
9	0.0005	0.0001010
12	0.0002	0.0000400
15	0.0001	0.0000275
18	0.0001	0.0000125
21	0.00001	0.0000040

IV. CONCLUSION

Space time block codes have been shown to be spectral efficient and have a high bit error rate. It is presumed that only the receiver has comprehensive channel information in a communication system. If this channel knowledge could also be used at the transmitter end, the entire system performance may be much improved. Several strategies for deploying this channel condition knowledge in terms of Channel State Information (CSI) at the transmitter have been presented in the literature. Some solutions rely on spectrally inefficient limited bit feedback systems, while others use code-words to describe channel conditions, which can then be communicated or used to change transmitted data based on the channel condition. This study compares the rotational precoder-based codebook technique to Space Time Block Coded and Orthogonal Space Time Block Coded systems with Zero Forcing and MMSE equalizers. The results suggest that using CSI at the transmitter end improves system performance. Furthermore, the precoded system based on STBC is the most spectrum efficient.

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