



# A Novel Approach for Channel Capacity Estimation of MIMO-OFDM using Water Filling Algorithm

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**Abstract**— This paper describes water filling algorithm which has been used for allocating the power to the MIMO channels so as to enhance the capacity of the MIMO network. Here we present a theoretical framework for allocating the power using 4x4 MIMO-OFDM system using Flat-fading channel. Different modulation technique is implemented under Nakagami channel, Rician and Rayleigh channel and BER performance is evaluated.

**Keywords**- BER, Nakagami channel, MIMO, OFDM, Rayleigh channel, Rician channel, Water Filling.

## I. INTRODUCTION

The growing demand on wireless communication services has created the necessity to support higher and higher data rate. Wireless communication systems face high level of ISI which originates from multipath propagation and inherent delay spread. A multipath based technique such as orthogonal frequency division multiplexing (OFDM) can be used to eliminate ISI and to improve capacity and spectral efficiency (bps/Hz) in wireless system [1]. In addition, MIMO systems are promising techniques to increase performance with acceptable bit error rate (BER) by using a number of antennas [2]. A MIMO-OFDM system transmits OFDM modulated data from multiple antennas at the transmitter. Data transmitted with subcarriers at different antennas are mutually orthogonal. The receiver extracts different data stream from different subcarriers after OFDM demodulation and MIMO decoding. Flat fading MIMO algorithms reduce computational requirement and make MIMO-OFDM attractive for mobile applications [3] With respect to single-input single-output (SISO) systems, multiple-

input multiple output (MIMO) systems over narrowband space-channels have been proved to be able to dramatically increase the spectral efficiency. However, narrowband MIMO systems severely suffer from the inter symbol interferences (ISI) over frequency-selective fading wideband channels. Orthogonal frequency division multiplexing (OFDM) is a multicarrier communication technique that is widely used, for e.g. in system such as Wi-Fi (802.11a/g/n) and WiMax (802.16).

Orthogonal frequency division multiplexing (OFDM) techniques transmit high rate data-stream over numerous sub-channels in frequency-domain and each sub-channel is a narrowband flat-fading channel. Thus, MIMO-OFDM systems have the potential to achieve high spectral efficiency over wideband channels, i.e., achieve high capacity. To achieve a high system capacity for multimedia applications in wireless communications, various methods have been proposed in recent years. Among them, the multiple input – multiple output (MIMO) system using multiple antennas at both the transmitter and the receiver has attracted a lot of research interest due to its potential to increase the system capacity without extra bandwidth. Multiple input-multiple-output (MIMO) exploits spatial diversity by having several transmit and receive antennas.

Previous work has shown that the system capacity could be linearly increased with the number of antennas when the system is operating over flat fading channels. In real situations, multipath propagation usually occurs and causes



the MIMO channels to be frequency selective. To combat the effect of frequency selective fading, MIMO is generally combined with orthogonal frequency-division multiplexing (OFDM) technique. OFDM transforms the frequency-selective fading channels into parallel flat fading sub channels, as long as the cyclic prefix (CP) inserted at the beginning of each OFDM symbol is longer than or equal to the channel length. The channel length means the length of impulse response of the channel as discrete sequence. The signals on each subcarrier can be easily detected by a time-domain or frequency-domain equalizer. Otherwise the effect of frequency-selective fading cannot be completely eliminated, and inter-carrier interference (ICI) and inter-symbol interference (ISI) will be introduced in the received signal. Equalization techniques that could flexibly detect the signals in both cases are thus important in MIMO-OFDM systems.

## II. PROPOSED METHODOLOGY

### A. MIMO-OFDM

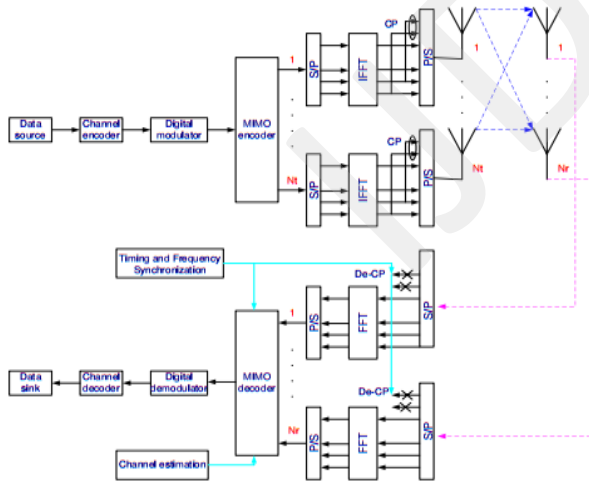


Figure 1: Block diagram of MIMO-OFDM

Equation shows expression for a MIMO-OFDM system with T transmit and R receive antennas, the received signal at the k-th sub-carrier of the n-th block from the j-th receive antenna:

$$Y_i = \sum_{i=1}^T H_{ij}[n, k] x_i[n, k] + \omega_j[n, k]$$

for  $j = 1, \dots, R$  and  $k = 0, \dots, K - 1$ , where  $x_i[n, k]$  is the symbol transmitted from the i-th transmit antenna at the k-th subcarrier of the n-th block,  $H_{ij}[n, k]$  is the channel's frequency response at the k-th sub-carrier of the n-th block corresponding to the i-th transmit and the j-th receive antenna, and,  $\omega_j[n, k]$  is additive (complex) Gaussian noise [4].

### B. Water filling

Water filling is the solution of several optimization problems related to channel capacity. The well known water filling algorithm solves the problem of maximizing the mutual information between the input and output of a channel.

The receiver can gain knowledge about the channel by the use of a known training sequence but if the transmitter should know anything about the channel it is necessary to use a feedback channel. The feedback channel consumes bandwidth in the channel or alternatively the capacity will decrease.

When the transmitter knows the eigenvalues and eigenvectors corresponding to the H matrix and the noise power ( $\sigma^2$ ) it can use this information to transmit in a smarter way. See [5] for an excellent description of the "Water filling technique". The Water filling technique is used to determine the powers  $\rho_k$  transmitted in each channel to achieve to greatest possible capacity. Consider a MIMO communication link with a shared total power budget of  $P_T$ , the capacity is then

$$C = \sum_{k=1}^n \log_2 \left( 1 + \frac{\rho_k}{\sigma^2} \lambda_k \right)$$

To achieve the greatest possible capacity  $\rho_k$  should be chosen in such a way that for every mode k

$$\rho_k = \left( \mu - \frac{\sigma^2}{\lambda_k} \right)^+$$

Where  $(x)^+ = \max(0, x)$  and  $\mu$  is the "water level".



Furthermore  $\mu$  should be chosen such that the total power budget is not exceeded, that is

$$\sum \rho_k = P_T$$

Important note: To obtain the optimal capacity the transmitter must have perfect knowledge of the H matrix (the eigenvalues and eigenvectors of H) and  $\sigma^2$ .

There will not be a general proof of the Water filling technique, but the idea will be shown for a 2\*2 MIMO system. The Method of Lagrange Multipliers will be used,

Maximize  $f(\rho_1, \rho_2)$  where,

$$f(\rho_1, \rho_2) = \log_2 \left( 1 + \frac{\rho_1}{\sigma^2} \lambda_1 \right) = \log_2 \left( 1 + \frac{\rho_1}{\sigma^2} \lambda_1 + \frac{\rho_2}{\sigma^2} \lambda_2 + \frac{\rho_1 \cdot \rho_2}{\sigma^2 \cdot \sigma^2} \lambda_1 \lambda_2 \right)$$

Under the power constraint,

$$g(\rho_1, \rho_2) = \rho_1 + \rho_2 \cdot P_T = 0$$

Which is an equivalent problem with Maximize

$$f(\rho_1, \rho_2) = 1 + \frac{\rho_1}{\sigma^2} \lambda_1 + \frac{\rho_2}{\sigma^2} \lambda_2 + \frac{\rho_1 \cdot \rho_2}{\sigma^2 \cdot \sigma^2} \lambda_1 \lambda_2$$

Under the constraint,

$$g(\rho_1, \rho_2) = \rho_1 + \rho_2 \cdot P_T = 0$$

Since  $\log_2(\dots)$  the function is monotonic.

Let

$$L(\rho_1, \rho_2, \nu) = 1 + \frac{\rho_1}{\sigma^2} \lambda_1 + \frac{\rho_2}{\sigma^2} \lambda_2 + \frac{\rho_1 \cdot \rho_2}{\sigma^2 \cdot \sigma^2} \lambda_1 \lambda_2 + (\rho_1 + \rho_2 - P_T) \nu$$

For critical points we want

$$0 = \frac{\partial L}{\partial \rho_1} = \frac{\lambda_1}{\sigma^2} + \frac{\rho_2}{\sigma^2 \cdot \sigma^2} \lambda_1 \lambda_2 + \nu$$

$$0 = \frac{\partial L}{\partial \rho_2} = \frac{\lambda_2}{\sigma^2} + \frac{\rho_1}{\sigma^2 \cdot \sigma^2} \lambda_1 \lambda_2 + \nu$$

$$0 = \frac{\partial L}{\partial \nu} = \rho_1 + \rho_2 - P_T$$

$$\text{Equation (A)} \Rightarrow \rho_2 = -\nu \frac{\sigma^2 \cdot \sigma^2}{\lambda_1 \lambda_2} - \frac{\sigma^2}{\lambda_2}$$

$$\text{Equation (B)} \Rightarrow \rho_1 = -\nu \frac{\sigma^2 \cdot \sigma^2}{\lambda_1 \lambda_2} - \frac{\sigma^2}{\lambda_1}$$

But since  $\nu$  is a variable which can be chosen arbitrary, a substitution can be made without any loss of generality

$$\mu = -\nu \frac{\sigma^2 \cdot \sigma^2}{\lambda_1 \lambda_2}$$

And since  $\rho_1, \rho_2 \geq 0$  must we have constraints on the choice of  $\mu$ . In the end we get that by choosing  $\mu$  in a proper way that satisfies and an optimal capacity is achieved.

$$P_T = \left( \mu - \frac{\sigma^2}{\lambda_1} \right)^+ + \left( \mu - \frac{\sigma^2}{\lambda_2} \right)^+$$

Accordingly to [6] the water-filling technique gives three different kinds power allocation depending on the SNR.

- *Low SNR*

At low SNR the Water filling technique finds the largest eigenvalues to H and sends the entire power through one single mode (channel). At this level of SNR the increase of capacity is almost linear and increases with 1bit/s/Hz for every 3dB increase of  $P_T$  power.

- *Intermediate SNR*

At intermediate SNR the water-filling technique uses L number of modes where  $1 < L < \min(n_T, n_R)$ . At this level of SNR the capacity is almost linear and increases with L bit/s/Hz for every 3dB increase of  $P_T$ .

- *High SNR*

At high SNR the water-filling technique uses all  $\min(n_T, n_R)$  modes for transmission. At this level of SNR the capacity is almost linear and increases with  $\min(n_T, n_R)$  bit/s/Hz for every 3dB increase of  $P_T$ .

### C. Rayleigh fading

Rayleigh fading is caused by multipath reception. The mobile antenna receives has large number of, say N, reflected and scattered replicas of same signal. Because of constructive and destructive interference, the instant received power seen by mobile antenna becomes a random variable, dependent upon the site of the antenna. The



probability distribution function of phase and amplitude of this random variable is given by:

$$p(R) = \frac{R}{\sigma^2} \exp\left(-\frac{R^2}{2\sigma^2}\right)$$

#### D. Rician fading

Rician fading is similar to Rayleigh fading except for the fact that there exists a strong line-of-sight component along with reflected waves. Redefined Rician models also consider

- That the dominant component can be a Phasor sum of two or more dominant components for e.g. the line-of-sight, plus a ground reflection. This joint signal is then mostly treated as a deterministic process.
- That the dominant wave can be subjected to the shadow attenuation. This is a popular supposition in the modelling of satellite channels.

Besides the leading component, mobile antenna receives a large number of reflected and scattered waves. The PDF of the amplitude can be determined as:

$$f_p(\rho) = \frac{\rho}{\sigma^2} \exp\left\{-\frac{\rho^2 + C^2}{2\sigma^2}\right\} I_0\left(\frac{\rho C}{\sigma^2}\right)$$

Where  $I_0$  is modified Bessel function of First kind and zero order.

#### E. Nakagami Fading

Nakagami Fading occurs for multipath scattering with relatively larger time-delay spreads, with different clusters of reflected waves. Within any one cluster, the phases of individual reflected waves are random, but the time delays are approximately equal for all the waves. As a result the envelope of each cluster signal is Rayleigh Distributed. The average time delay is assumed to differ between the clusters. If the delay times are significantly exceed the bit period of digital link, the different clusters produce serious inter-symbol interference. The Nakagami Distribution

termed the magnitude of the received envelope by the distribution

$$p(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega_p}\right)^m r^{2m-1} \exp\left\{-\frac{mr^2}{\Omega_p}\right\}$$

### III. RESULT

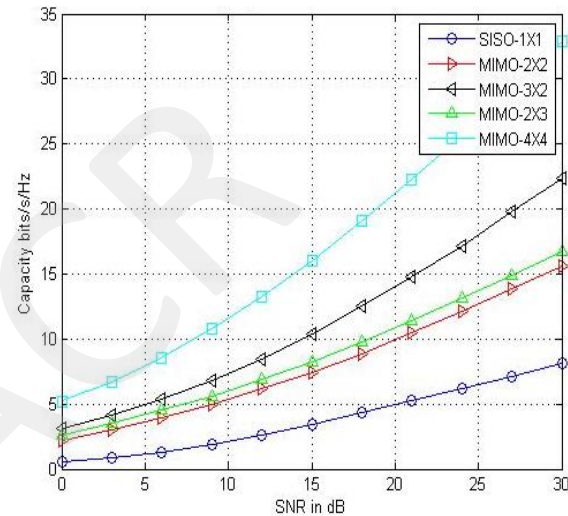


Figure 2: Channel capacity estimation for Nakagami channel

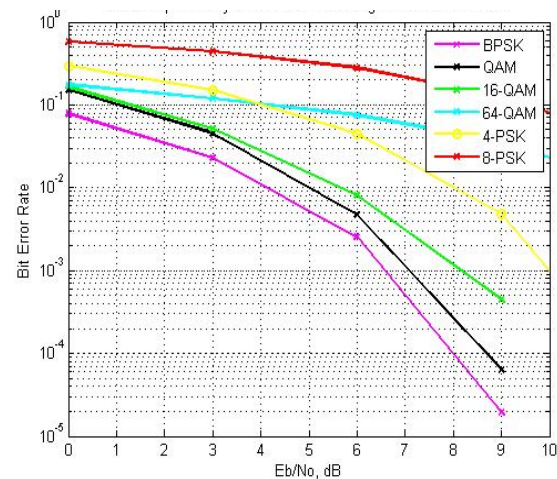


Figure 3: Bit error probability curve for OFDM using different modulation

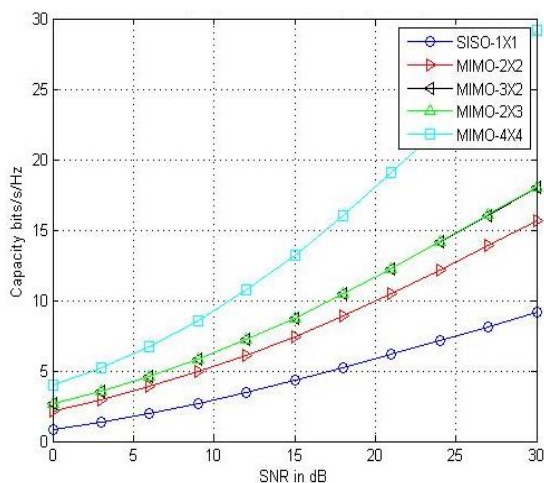


Figure 4: Channel capacity estimation for Rayleigh fading channel

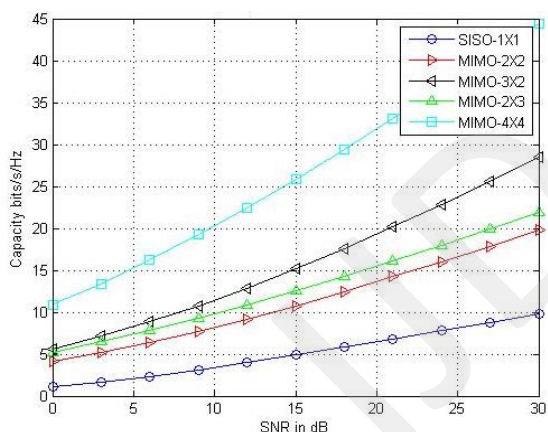


Figure 5: Channel capacity estimation for Rician channel

The above simulated results depicted shows the channel capacity v/s SNR where the capacity increases according to increase of transmit diversity, as antenna size is 4\*4 then its capacity is maximum.

#### IV. CONCLUSION

In this paper, we have presented the MIMO OFDM model using MATLAB. The results of simulation form the model will enable the researches to choose water filling algorithm for their requirements. MIMO has helped to ISI problem. The Results indicates that the Capacity

is enhanced significantly by transmitting the data through different channels. The simulation results shows that water filling algorithm give enhanced results for Rician channel

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